Psychological Review

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THE PSYCHOLOGICAL REVIEW

A THEORY OF DATA

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INTRODUCTION

The behavioral sciences in general and psychology in particular are laden with methods for collecting and analyzing data. These methods usually have names associated with them which do not always clearly imply whether they are methods for collecting data, or for analyzing data, or for both. Thus the method of pair comparison is a method for collecting data, the law of comparative judgment (Thurstone, 1927) is a model for analyzing data, but psychophysical methods imply both. It is probably generally true that a method for analyzing data implies certain conditions that must be met by the method of collecting the data but there are many variations in the methods of collecting data that may satisfy the same conditions. When these conditions become clear the full generality of the methods for analyzing data becomes apparent. This generality is obscured by terminology particular to a context as in psychophysics or attitude scaling and it would seem desirable to abstract the properties of all methods and see thereby what is common among them and how they differ.

The theory of data proposes to do this. It proposes to provide a foundation for models of psychological measurement and classify, systematize, and interrelate them. It is by no means proposed that this is the only schema or the best one.

The domain of discourse of the theory of data includes the methodologies in what psychologists speak of as the areas of psychophysics, mental testing, attitude scaling, latent structure analysis (Green, 1954), scalogram analysis (Green, 1954), preferential choice behavior, rating scales, factor analysis, multidimensional psychophysics (Torgerson, 1958), etc.

Many of the psychologist's methodologies have been constructed with a particular content in mind such as mental testing or attitude scaling and hence are identified with content areas and use the vocabulary of such content. Courses in these various methodologies are frequently content oriented and the student may not be aware of the identities and differences among them. When such content-oriented models are cast in abstract form they are recognizable as miniature behavior theories, the scope of their applicability

is broadened, and alternative theories immediately spring to mind. There is perhaps less of a tendency to feel "this is the way to analyze that kind of data."

Initial steps toward a theory of data were first published in 1952 (Coombs, 1952) and a more explicit formulation of a theory of data a year later (Coombs, 1953). This current formulation, while intimately related to the previous, is a thorough revision and, I hope, a vast improvement.

The next three sections describe, in turn, the theory of data, the application of the theory to the classification of models and their relations within classes, and finally, a general discussion of the classes of psychological data and some of the relations between classes. At the end is a brief glossary and a mathematical appendix.

THE THEORY OF DATA

The fundamental ingredients of those psychological observations to which measurement models are applied are surprisingly simple. From the point of view of measurement models, basically all a person can do is compare stimuli with each other or against some absolute standard or personal reference point, the stimuli may come singly or in pairs, and the comparison is one of dominance or consonance. For example, an individual passing or failing an arithmetic problem may be regarded as a comparison between his ability (a personal reference point) and the problem's difficulty (a stimulus) and the comparison is one of dominance.

Before illustrating these basic ingredients of behavior it will be useful to abstract them explicitly. This is done by the fourth axiom (see Appendix) as follows:

Every measurement model may be regarded as satisfying each of three dichotomies:

(a) A relation exists on a pair of points or on a pair of pairs of points.
(b) The elements of a pair of points are drawn from two distinct sets (A) or from one set (B).
(c) The relation is either an order relation ($>$) or a proximity relation (O).

Consider the following two illustrations:

(a) An individual endorses an attitude statement. A model might regard the attitude of the individual as a point drawn from one set of points in a psychological space, and the attitude statement as a stimulus point drawn from another distinct set of points in the same space, and the observation ("endorses") as a proximity (O) relation on this pair of points. That is, that the stimulus point is "near" or "in the neighborhood of" the point corresponding to the attitude of the individual. It is, of course, by no means implied that this is the necessary model for that observation, but rather that it is a "theory" as to how this behavior is generated.

(b) An individual says that one pair of color patches are more alike than another pair. A model might regard each pair of stimuli as a pair of points and the four points contained in the two pairs of points as being drawn from the same set, and the observation (more alike) as signifying that the "distance" in the psychological space between one pair of points is greater than ($>$) the distance between the other pair of points.

This 4th axiom is the critically significant axiom for the theory of data but clearly some prior mathematical machinery is necessary—so we begin at the beginning.

The essential objective of every psychological measurement model is to associate with each object of interest, individual or stimulus, a point in a psychological space, and the purpose of the model is to construct a calculus which will permit the recovery of the space, given the observations and the preconceptions of the space. A great variety of preconceptions of the prop-

erties of a psychological space are to be anticipated so the postulates of the theory of data are very general in order to accommodate this variety.

The four sets D, H, I, and J given in the appendix are merely label sets. The set D is for designation of dimensions and the set H for designation of trials—a quantized temporal variable. The sets I and J are for the designation of two distinct sets of objects. An illustration might be a set of individuals and a set of stimuli, in which case we shall adopt the convention of using the label set I for individuals and the label set J for stimuli. It is by no means necessary, however, that this always be the case. Sometimes, for example, individuals may be used as stimuli in which case the label set J would be used. Furthermore, some models label only the stimuli but deal with two distinct sets of stimuli, in which case the label sets I and J will be used.

As stated previously, the purpose of a model is to recover a psychological space given the data. Thus one model is concerned with scaling statements of opinion on a one dimensional continuum (that is, locating points on a line) and another model is concerned with the number of dimensions characterizing a set of statements of opinion and locating points in a multidimensional space which will account for the behavior observed. Hence the first axiom with its accompanying definition simply postulates the existence of such a space, with r dimensions and each point an r-tuple.

Note that the axiom says each dimension "is a segment of the real line," that is to say, inherently a ratio scale. This is quite a strong statement and could lead to endless interesting and futile philosophical argument. It is quite true that for some models it is sufficient to postulate merely an ordinal scale for the elements of a dimension K^d but

other models would require an ordered metric, others an interval, and others a ratio scale. Rather than a series of successive versions of this axiom strengthening it as necessary, I chose to assume as much as is necessary to accommodate all the models, and then one may speak of weaker measurement models as "recovering" this space at lower levels of scales.

Measurement models variously identify points in this space K with stimuli and/or individuals so it is convenient to construct some sets of points in the space K. Two subsets of points are constructed which are called C and Q. C is the subset of points which are labelled by the I set and Q is the subset of points labelled by the J set. It might be well to point out that while the sets I and J are label sets for two distinct sets of objects (in the real world) it is not necessary that the subsets of points C and Q be distinct. For example, one might conceive of a statement of opinion that precisely reflects how an individual feels and while it is desirable to distinguish between the individual and the statement of opinion there is no necessary distinction between the points in the psychological space that correspond to them.

Having the two sets of points C and Q it is useful to construct sets of pairs of points. Consequently we conceive of a set of pairs of points where one is a member of the set C and the other a member of the set Q and call such a set the set A of ordered pairs (c_i, q_j). In accordance with the convention established, such a pair of points might correspond to an individual and stimulus, respectively, or a pair of stimuli from two sets as the two lines in a Mueller-Lyer illusion which terminate in a feather or arrow.

Sometimes the observations are made on pairs of such pairs where the same individual enters into both pairs, as, for

example, when an individual is asked which of two statements he prefers to endorse. So it will be convenient to construct a subset of the set A, consisting of those pairs of points (c_i, q_j) where i is fixed, such a subset is labelled A_i. The subset A_i then consists of pairs of elements, one a fixed individual i, and the other a stimulus j.

For some models the members of a pair of points are drawn from the same set. It makes no difference whether such a pair is regarded as being drawn from the set C or the set Q but inasmuch as such pairs of points are usually identified with stimuli they will be regarded as drawn from the set Q and the set of such pairs of points drawn from the same set is called B. A typical example is in the scaling by pair comparison of lifted weights or brightness of lights.

It is interesting to note that when models deal with observations on pairs of pairs of points the pairs of points are always drawn from the same set, A or B, never is one drawn from A and the other from B. There is nothing logically necessary about this, of course, it is just that there are no psychological measurement models specifically constructed for such data.

The second axiom postulates the existence of a "distance" function in the space K—that is, between every pair of points in the space K there is a "distance" between them. It is to be noted that nothing is said about this distance concept other than that it satisfies the minimum conditions for a distance function. It is not required, for example, that the space be Euclidean or any other particular geometry. Some models do not require a metric space at all and may be spoken of as "recovering" the space at a lower level.

The third axiom merely links the first two together and defines a positive direction for each dimension K^d.

The fourth axiom has already been discussed, so we conclude this section with a discussion of the fifth axiom and its accompanying definition. On a given trial or moment (h) when the behavior is interpreted as a relation between a pair of points from distinct sets (conveniently referred to as an individual, i, and a stimulus, j) we conceive of the behavior as being generated by some but not necessarily all of their attributes. Hence, the distance between the pair of points is a distance in a subspace of the total space called *the relevant dimensions, D'*.

In a similar manner, if the behavior of an individual (i), on a given trial or moment (h) is interpreted as a relation on a pair of points from the same set (the stimuli j and k), we conceive of the behavior as being generated by some but not necessarily all of the attributes in the space K. Hence, the distance between the pair of points j and k is a distance in the subspace called *the relevant dimensions, D"*.

The discussion of these axioms and their accompanying definitions complete the description of the basis of the theory of data. We proceed next to construct the eight types of data and the definition of the information in each. The objective here is to illustrate the mapping between this abstract model on the one hand and the types of observations made by psychologists.

THE CLASSIFICATION OF PSYCHOLOGICAL MEASUREMENT MODELS AND METHODOLOGIES

Taking the conjunction or cross-partition of the three dichotomies given in Axiom 4 yields eight classes—a cube which is $2 \times 2 \times 2$. For simplicity of portrayal it may be drawn in a plane with one of the three dichotomies projected onto the other two. I have chosen to project the third because this dichotomy is collapsed when an inter-

mediate category of response of the form "I can't decide" is used. This point will be made clear in the discussion of the kinds of data below.

The distinctions of the theory of data may be portrayed, then, as in Fig. 1.

The four classes generated by the cross-partition of the first two dichotomies have been labelled as quadrants I to IV, and the additional distinction, an order ($>$) vs. a proximity (O) relation, divides each quadrant into an a or b octant, respectively. In each octant the set from which the points are drawn is indicated in the figure.

In general, for all the octants, a quantity p is assumed to exist which is a distance between a pair of points, and an observation is made concerning the magnitude of such p's or pairs of such p's. The precise definition of this information is different for each octant and they are all given in the appendix. The measurement problem may be simply put as the problem of how to construct the space K given the information about the p's. Any psychological measurement model constitutes a calculus for that purpose, adding axioms to those given here, such as specifying that the dimensionality of the space K is *one*, for example, or for a dimensionality greater than one, specifying that the distance function is Euclidean or otherwise (e.g., cf. Householder & Landahl, 1945, Ch. 8 & Attneave, 1950). It is clear from this why psychological measurement has so often been spoken of as distance measurement (Bentley, 1950).

In the remainder of this section the quadrants will be discussed in turn, first the two octants separately and then the quadrant as a whole. Examples of the kinds of behavior that could be mapped into each octant are given with their corresponding interpretations as data in the form of relations on points. A brief summary of the models avail-
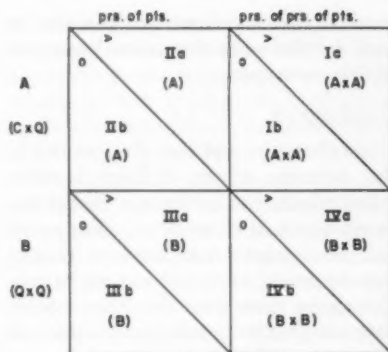
FIG. 1. Distinctions of the theory of data.

able for the data of each octant is also presented.

Quadrant Ia

An illustration of behavior which might be mapped into this octant is the behavior of an individual when asked which of two alternatives he prefers. When such behavior is so mapped it is assumed that there is a point corresponding to the individual representing an "ideal" point and each alternative also corresponds to a point and the individual will prefer that alternative "nearer" to his ideal point. Such data are order relations on pairs of distances.

The only currently available model for analyzing Quadrant Ia data is the unfolding technique (Coombs, 1950, 1952; Bennett, 1951; Hays, 1954). Analysis of such preferential choice data by the unfolding technique yields a joint space (in one dimension it is called a *J scale*) in which are located points associated with individuals and points associated with the stimuli. This is a genotypic structure. The development of the method for Euclidean multidimensional spaces is due to Bennett (1951) and Hays (1954). Papers presenting this development are now in press. More recently Coombs and Kao (in press) have shown how the tech-

Quadrant Ib

An illustration of such data as this is the behavior of an individual when asked whether or not he can choose between a pair of alternatives. For every pair of alternatives he answers merely that he would or would not prefer one alternative more than the other. Such data are proximity relations on pairs of distances. There are no models specifically designed to analyze such data, perhaps because the amount to be learned from such data is small compared with Quadrant Ia data. The primary purpose in mentioning it here, aside from formal completeness, is its relevance to the interpretation of preferential choice data when an intermediate category of judgment is used. Such data constitute what might be called Quadrant I in that there are data in each of the two octants.

Quadrant I

If an individual is asked which of two alternatives he prefers and is permitted to answer "I don't know" or "I can't say" we may speak of this as using an intermediate category of judgment.

The result would be that when an individual made a choice the data would be Quadrant Ia and when he failed to make a choice it would be Quadrant Ib. No models exist for the analysis of such data to yield a joint space; one would be compelled to neglect responses using the intermediate category of judgment or to make assumptions which would map it into Quadrant Ia data for which models are available.

Quadrant IIa

This is the most prevalent kind of psychological data so a number of examples of it will be given. One illustration of this type of data is mental test behavior—for example, the response of an individual to an arithmetic item. An individual passing an item implies that he has more of the ability involved than is required by the difficulty of the item. The phenotypic behavior, pass or fail, is interpreted as an order relation on a pair of points—one the individual's ability and the other the difficulty of the item.

In another context, the study by Janis (1949) on psychophysiological correlates of fear is interpreted as this type of data. When the individual said "yes" or "no" to having experienced a particular fear symptom in combat, this was interpreted as an order relation on a pair of points, one corresponding to the amount of fear he had experienced and the other corresponding to the amount implied by the symptom.

Responses of individuals to neurotic inventory type items are typically interpreted as this kind of data also. An individual answers "yes" or "no" to the question "Do you wet your bed frequently?" The answer is interpreted as an order relation between a pair of points, one corresponding to the individual's perception of how frequently he wets his bed and the other corresponding to his decision as to how often one must wet his bed to do it "frequently."

Psychophysical threshold studies deal with identically equivalent data in a formal sense. When an individual is asked whether or not he perceives a stimulus, the behavior is interpreted as an order relation on a pair of points one corresponding to his threshold and the other corresponding to the magnitude of the stimulus. In the case of determining a difference limen, the same thing holds if the stimulus is defined to be a *difference* between two stimuli.

Clearly the data of psychophysical

studies, mental tests, and neurotic inventories all have the same formal character of consisting of order relations on pairs of points from distinct sets. But certain further differences remain. The great advantage that psychophysics has over mental testing is the apparent possibility of experimentally independent replication of the individual-stimulus pair. The great advantage that mental testing has over that of the neurotic inventory is that the individual is not able to decide for himself how difficult an arithmetic problem is. The difficulty of an arithmetic problem is presumably more compelling than how shy you have to be to say yes. In abstract terms, the point corresponding to the difficulty of an arithmetic item is, presumably, relatively stable over different individuals compared with the point corresponding to the item, "Are you shy?" But in a formal sense the data from these separate areas are all equivalent, the same basic measurement models apply, and the concepts and problems of one suggest equivalences in the other.

There are a variety of methods and models for the analysis of this kind of data. These include Guttman's scalogram analysis (Green, 1954), Lazarsfeld's latent distance model (Green, 1954), test theory (Gulliksen, 1950), the law of categorical judgment (Torgerson, 1958), Coombs-Kao nonmetric factor analysis models (1955), and multiple factor analysis (Thurstone, 1947).

These models all yield a joint space in which the elements of two distinct sets are located, typically individuals and stimuli, except in the case of the law of categorical judgment, as explained below. They vary in their assumptions and hence correspond to different theories of how the behavior is generated. Some models, like test theory yield a one-dimensional J scale

no matter how the individuals behave, and then measures of reliability and homogeneity are constructed which are related to how good a fit is obtained by the one dimension. Another model, like scalogram analysis, yields one dimension but is highly sensitive to imprecision or lack of unidimensional homogeneity. This method is essentially a method for testing whether certain conditions for unidimensionality in the genotypic structure generating the behavior are met. One has one's choice here between models which are deterministic or probabilistic and which are unidimensional or multidimensional.

Torgerson's law of categorical judgment is designed to analyze the data obtained by having individuals sort stimuli into ordered classes and the occurrence of a stimulus in a particular class is interpreted as an order relation between a point corresponding to the stimulus and a point corresponding to one of the boundaries of the class. Here the two distinct sets of real world objects are the stimuli being sorted and the boundaries of the classes.

Quadrant IIb

Typical of the kind of behavior mapped into this octant is that of an individual agreeing or disagreeing to endorse a statement of opinion. Such behavior is interpreted as a proximity relation on a pair of points, one corresponding to the attitude of the individual, the other corresponding to the attitude expressed by the statement of opinion. More broadly, the interpretation is that there are two sets of elements, and the elements of one set are being matched with the elements of another set. Clearly, in the most general case, a labelling process. Membership in an organization, marriage, and clinical diagnosis are all further examples of matching between elements of distinct sets.

A classic experiment that may be used to illustrate the data of Quadrant IIb is the study of Watson and Watson (1921) of the generalization of a conditioned fear response to a white rat. The child, Albert, having been conditioned to fear a white rat, may be represented by a point in a psychological space and the occurrence of a conditioned response to other stimuli, such as a white rabbit, (represented by another point in the space) reflects a proximity relation on the pair of points.

In all of these cases there are two sets of points and the elements of one set are matched with the elements of another set. In the case of endorsing attitude statements there is a set of points corresponding to individuals and another set of points corresponding to statements of opinion and the individuals match themselves with the statements. But also it should be eminently clear that as far as the formal aspects of the data are concerned it doesn't matter who does the matching. Thus, rating scale behavior, magnitude estimation, and magnitude production are also illustrations of this kind of data in that one has a set of objects to be judged and another set of elements which constitute the response categories and the objects of judgment are matched with response categories.

When next to nothing is known a priori about the relation between elements within the two classes, Lazarsfeld's latent class model is the most appropriate. A good illustration of this is in the study by Gibson of the preferences of radio listeners for 13 types of evening programs (Lazarsfeld, 1959). However, in certain cases, as in rating scales, the elements of one of the sets, the set of response categories, has an a priori order relation on the elements of the set. Thus, a foreman who is rated "superior" is presumed better than one rated "mediocre" and

by virtue of this order relation on the elements of one of the sets there is generated an order relation between pairs of elements from two sets. The consequence of this is that one has the option of analyzing such data by the models of Quadrant IIa. Thus rating scale data may be analyzed by the law of categorical judgment in which one does not need to assume the classes are equally spaced on the continuum, and a scale may be recovered in which both the stimuli and the boundaries of the classes are scaled.

If the elements of one of the sets are not only simply ordered but are numbers, as in some rating scales and in magnitude estimation, then numbers may be associated with the objects being judged and these numbers have the properties assumed to be true for the numbers constituting the response set.

Probably the oldest model for collecting and analyzing a special kind of data in Quadrant IIb is the method of average error as applied, for example, to measure the extent of the Mueller-Lyer illusion. The two distinct sets are the two sets of line segments, one with terminal arrowheads and one with terminal feathers. The elements of one set are matched vs. the elements of the other set. The method requires that the elements of both sets be measurable on the same physical scale and then the mean difference between matched elements is the extent of the illusion for that individual or collection of individuals.

Quadrant II

Just as in Quadrant I where an intermediate category of response generated data in both Quadrant Ia and Quadrant Ib, so also is this true of Quadrant II. The behavior would have to be of such a nature that if the point corresponding to the individual ex-

ceeded the point corresponding to the stimulus by more than an amount ϵ_{hij} he reacted positively, if by less than an amount $-\epsilon_{hij}$ he reacted negatively, and otherwise intermediately. Thus, if an individual, in evaluating candidates for office, had the following three categories of response available:

(a) No, he's too liberal
(b) Yes
(c) No, he's too conservative

such behavior could be interpreted as Quadrant II data including both IIa and IIb.

The most recent model constructed for analyzing this kind of data is Lazarsfeld's latent structure analysis. This is actually a very general model which can admit a mixture of monotone and nonmonotone trace lines and thus would be a model for Quadrant II as a whole.

Quadrant IIIa

A judgment as to which of two stimuli has more of some attribute is an example of the kind of behavior mapped into this octant. The behavior involves the comparison of stimuli, as in much of psychophysical scaling, and is interpreted as an order relation on a pair of points, both identified with stimuli. The distinction to be noted between Quadrant IIIa data and Quadrant Ia data is that in the latter the individual is also conceived of as being represented by a point in the space with the stimuli whereas in Quadrant IIIa data the individual is not represented by a point in the space. Athletic meets, such as tennis tournaments and professional baseball, are examples in which "nature" is making such a pair comparison (Mosteller, 1951).

The intent of all Quadrant IIIa data is to construct a subjective scale of stimulus magnitude and all of the models for analyzing Quadrant IIIa

data yield spaces in which only the elements of a single set are located. Hence, these models may be spoken of as yielding a stimulus space (in one-dimension it is called a stimulus *scale*) in contrast with the joint spaces which generate data in Quadrants I or II.

If the order relation on pairs of points is transitive an ordinal scale of the stimuli follows immediately. If there are replications on each pair of stimuli which yield a probability other than zero or one that one member of the pair is greater than the other, then an ordinal scale follows immediately if weak stochastic transitivity is satisfied (Davidson & Marschak, 1958). Models exist which made transformations of these probabilities into psychological distances on a subjective stimulus scale. Thurstone's law of comparative judgment (Thurstone, 1927) yields an interval scale and Luce's recent model for choice behavior (Luce, 1959) yields a ratio scale. Each of these require that the probabilities satisfy strong stochastic transitivity among other things. In S. S. Stevens' (1957) method of ratio estimation the individual judges not only which stimulus is greater but how many times greater and the result is a ratio scale also. On a certain primitive level the data to which these models apply is the same—order relations on pairs of points from the same set. Data collected by Stevens' method could be analyzed by either Luce's or Thurstone's model but not vice versa. So Stevens' method requires more information in the response than do the others.

Quadrant IIIb

The kind of behavior that represents data in Quadrant IIIb is the response of an individual as to whether two stimuli are the same or not—that is, whether they *match*. This type of data has only recently become of interest through the book of Goodman's (1951).

Galanter (1956) has since begun the construction of a model for analyzing such data. Even more recent is the work of Hefner (1958) on the construction of a model for this octant for data obtained by degrading stimuli through brief time exposures.

The potential significance of this type of data (Quadrant IIIb) resides in the fact that the symmetric predicate (do the stimuli match or not) yields information about the distance between a pair of stimuli in the stimulus space and hence may lead to the exploration of multidimensional stimulus spaces.

The data of Quadrant IIIa, in contrast, is derived from an asymmetric predicate (which stimulus is more of something) and there is serious question as to whether such data could lead to anything more than a one-dimensional stimulus scale. These issues are discussed fully by Goodman.

Clearly, if an experimenter wanted to construct a one-dimensional stimulus scale, an asymmetric predicate should be used. Whereas, if he wants to explore the cognitive space in which the perceptions of the stimuli are imbedded, the symmetric predicate of Quadrant IIIb is superior.

However, the construction of multidimensional spaces is also possible with the asymmetric predicate of Quadrant IVa and will be discussed below.

Quadrant III

Finally, as in the case of the preceding quadrants, if an individual in judging which of two stimuli is greater were permitted to respond "I can't decide" the behavior constitutes Quadrant III data and a finite ϵ corresponding to a threshold for decision would be involved.

It is of interest to note that sociometric matrices contain data belonging in this quadrant. If the matrix is asymmetric, as in who bosses whom,

the data are Quadrant IIIa; if the matrix is symmetric, as in whom do you go to the movies with, the data are Quadrant IIIb; and if the matrix is mixed, as in whom do you like, the data are Quadrant III. Thus once again we see relations between models apparently constructed for quite different purposes but now become suggestive for other real world content.

Quadrant IVa

The behavior of individuals when presented with two pairs of stimuli and asked which pair is more alike is representative of behavior typically mapped into Quadrant IVa. The basic observations are the comparative similarities of pairs of stimuli. The individual is presumed to be responding to the distance between the members of a pair of stimuli. Such data may lead via the unfolding technique to a one-dimensional ordered metric stimulus scale (Coombs, 1954a) if certain conditions are satisfied by the data. If one dimension will not satisfy the data then a multidimensional solution may be sought.

Hays, in a paper being prepared, has adapted his multidimensional unfolding solution of Quadrant Ia data to the data of Quadrant IVa and the result is a nonmetric model for multidimensional psychophysics which requires only an order relation on the distances between pairs of points in the space (assumed to be Euclidean). This model yields the stimulus space recovered only at the level of a product of simple orders. An example is contained in Coombs (1958).

Torgerson's model (1958) assumes more and yields more: (a) the order relations on distances are transformed into measures of the distances on a ratio scale by a Quadrant IIIa model (b) these distances between pairs of points are transformed into scalar prod-

ucts (*c*) which are then factor analyzed. The result is the recovery of a real Euclidean space.

Quadrant IVb

The type of behavior which would be mapped into Quadrant IVb would be the response of an individual to two pairs of stimuli which was interpreted to mean that one pair was no more alike (or different) than the other pair. Again, as in Quadrant IVa, the individual is presumed to be responding to the distances between the members of a pair and the response is interpreted as a proximity relation on these distances. Observations collected by the methods of equisection in psychophysics are representative of behavior mapped into this octant.

The data of this quadrant seem to be of only slightly more interest than that of Quadrant Ib.

Quadrant IV

If an individual were permitted to judge that one pair is more alike than the other or that he can't decide, the behavior would fall into both Quadrant IVa and IVb and would constitute Quadrant IV data again introducing a finite ϵ as a parameter.

GENERAL DISCUSSION

The Four Basic Kinds of Psychological Data

From the preceding discussion of the information in data, it is apparent that the dichotomy of whether an order relation or a proximity relation is observed is a subordinate dichotomy to the others in the sense that it is not satisfied when an intermediate category of response is used by the experimenter. Hence the four quadrants rather than the eight octants may be seen as representing four primary kinds of psychological distance observations. As a mnemonic convenience, these four quadrants may be given names to signify descriptively the type of behavior that is mapped into each.

In Quadrant I, the relation observed is on a pair of distances where each distance is between a pair of points from distinct sets, usually an individual and a stimulus. The real world context in which this kind of data is most commonly obtained is in observing the preferential choices of an individual over a set of stimuli. The data may be referred to as individual-stimulus differences comparison or, more meaningfully, *Preferential Choice* data.

In Quadrant II, the relation observed is on a pair of points which are from distinct sets, typically an individual and a stimulus. Such data may be called individual-stimulus comparison data, more commonly known in psychology as *Single Stimulus* data. It is important to note that this includes not only mental test data, endorsing statements of opinion, and psychophysical threshold data, but also rating scales, the method of successive intervals, magnitude estimation, and, in general what is known as absolute judgment data.

In Quadrant III, the relation observed is on a pair of points which are from the same set, called stimuli. So such data may be called *Stimuli Comparison* data. One is tempted to call methods for collecting such data psychophysical methods, but this would lead to confusion with psychophysical studies of thresholds which belong in Quadrant II. Also, such a name is content-bound and there are many examples outside of conventional psychophysics which deal with identically the same kind of data—as for example, sociometric matrices.

In Quadrant IV, the relation observed is on a pair of distances where each distance is between a pair of stim-

Single Stimulus Data or Individual-Stimulus Comparison	Preferential Choice Data or Individual-Stimulus Differences Comparison
Stimuli Comparison Data	Similarities Data or Stimuli-Differences Comparison

FIG. 2. The four quadrants.

uli. This kind of data has led to the construction of models under the rubric of multidimensional psychophysics. The data could be called stimuli-differences comparison. Inasmuch as the behavior is typically a response to the relative similarity of stimuli, the name *Similarities* data is proposed.

Figure 2 contains the four quadrants with their suggested labels indicated.

Some Interrelations of Quadrants

It is important to note that there is no unique mapping of behavior into these quadrants. An experimenter, when he analyzes his data, has made a choice of a behavior theory. He has decided, for example, that the behavior was generated in a space in which both the individuals and stimuli were points or just the stimuli are points, he may decide that the behavior is generated by one-dimension or he may ask if it is in a space of more than one dimension. If he decides that the behavior may be generated in a space of more than one dimension, he is faced with deciding what kind of a distance function to employ.

On this latter point, the use of a Euclidean distance function was almost universal until very recently. Perhaps the earliest thoughts of alternative distance functions occur in a paper of H. M. Johnson's (1935) and these ideas are more fully developed in the conjunctive and disjunctive models of the Coombs-Kao nonmetric factor analysis monograph. Another, perhaps very significant, alternative distance function is suggested by Householder and Landahl (1945) and has been picked up by Attneave (1950) and applied to the area of perception. Once the barrier of convention is broken down, one may expect a great variety of possible distance functions to be developed, each corresponding to a theory about how complex behavior is generated. Distance functions conceived of in the context of data in one quadrant suggest the construction of equivalent models for other quadrants.

To illustrate the kind of decisions an experimenter makes in analyzing data, suppose he has the pair comparison preferences of each of a number of individuals over a set of alternatives. Such behavior has been used here to illustrate data in Quadrant Ia, which, when analyzed by the unfolding technique, leads to a joint genotypic space with both individuals and stimuli located in it and the dimensions of this space correspond to the latent attributes generating the individuals' preferences.

One may recognize here, however, that the individual is making a pair comparison between distances, a distance being the distance between the point corresponding to him and the point corresponding to the alternative. The experimenter may decide then, that the *distance* is the stimulus and each individual's behavior may be interpreted as an order relation on a pair of stimuli which are these distances. This *distance* for each alternative is how much the individual dislikes each alternative, i.e., the further the stimulus point from the ideal point

the less it is liked. The experimenter then, may decide that he wishes to construct a stimulus scale for the alternatives, representing a measure of their preferability. Consequently, he maps the behavior into Quadrant IIIa and scales the alternatives, say by the law of comparative judgment. One obtains, then, a scale with only the alternatives on it ranging from most to least preferred, representing an amalgamation of the individuals' preferences. The interpretation of such a scale and its relation to the results obtained by analyzing the same behavior as Quadrant Ia data are discussed in Coombs (1952, 1954b).

This serves to introduce the more general case in which behavior mapped into Quadrants I and IV may always be so interpreted as to be mapped into Quadrant III. This comes about in this way. In Quadrants I and IV, a comparison is being made between distances—in Quadrant I it is a distance between an individual and a stimulus (or more generally, a distance between points in distinct sets) and in Quadrant IV, it is a distance between stimuli (or more generally a distance between points in the same set.) If, however, the experimenter chooses to regard these *distances* as the *stimuli*, then the behavior maps into Quadrant III, with Ia and IVa going into IIIa, and Ib and IVb going into IIIb.

That this commonly occurs with Ia data in the construction of scales of preference has already been discussed above. This also occurs in multidimensional psychophysics as a preliminary step in which a scale of the *distances* between pairs of stimuli is first constructed and then these distances are analyzed by the methods of multidimensional psychophysics to recover a space in which the original stimuli may lie.

These distances between stimuli, however, may be initially obtained by other methods, such as rating scales (Quadrant IIb) or the method of categorical judgments (Quadrant IIa). These are the approaches used by Ekman (1954) and Mellinger (1956) respectively.

The psychophysical methods represent models which apply to data in more than one quadrant. They apply to data which fall into Quadrant IIa or IIIa, depending on whether the experimenter conceives of the behavior as reflecting a measure of the individual (as in threshold determination) or as reflecting measures of the stimuli. These are respectively IIa and IIIa data.

Other Response Measures

The point has been made that psychological observations are interpreted as relations on points or in equivalent terms, as distance measurements. The psychological observations which have been used for illustration have been for the most part judgmental responses but these are by no means the only kind of observations that are made nor the only kind of distance measurement. Other kinds of response measures which are used to generate data are observations of inconsistency of response, latency, amplitude, and confusion errors.

These response characteristics or measures may also be interpreted as relations on points and hence yield psychological data to which psychological measurement models may be applied. There appears to be an interesting difference between the psychological data obtained from such response measures as these and that obtained from judgmental responses. The former seem to be interpretable as information only about the *absolute* distances between pairs of points whereas the latter may yield information about either *absolute* or *algebraic* distances. The more often

an individual confuses two stimuli, the longer it takes him to choose between them, the less the distance between their respective points in the psychological space. These response measures do not appear to have information in them as to which stimulus is on which side of the other, i.e. the algebraic distance. Models which make use of such data have been generally concerned with the kind of transformation that should be made to reflect a measure of psychological magnitude. Thus, there are the models of Thurstone (1927) and Luce (1959) for the transformation of inconsistency into psychological distance. Both apply to data mapped into Quadrant IIIa and yield measures of the stimuli. Less has been done in a formal way to construct models for the other kinds of response measures. The difficulty is that there is little experimental literature which is of aid in suggesting the kinds of assumptions one can make for transforming the response measure into a psychological distance. What is necessary is some intensive experimental work on these fundamental aspects of psychological measurement.

Relation to Older Theory of Data

The first explicit form of the theory of data (Coombs, 1953) constructed four quadrants on the basis of behavior being interpreted as relative or irrelative and as Task A or Task B. In one of the four quadrants so generated, the one corresponding to single stimulus data, there was a further dichotomy based on whether the stimuli were monotone or nonmonotone (also referred to as cumulative and noncumulative). The mapping between the old and new form of the theory is indicated in Table 1.

The old Quadrant III is imbedded in the new Quadrants IIa and IIb and the old Quadrant IV has been differenti-

TABLE 1

RELATION BETWEEN OLD AND NEW THEORY OF DATA

Old	New
Quadrant I	Quadrant Ia
No Equivalent	Quadrant Ib
Quadrant IIa, Quadrant III	Quadrant IIa
Quadrant IIb, Quadrant III	Quadrant IIb
Quadrant IV	Quadrant IIIa, IIIb, IVa, IVb

ated into four octants. The old Quadrant III would include all methods involving the evaluation of stimuli, one at a time with respect to an attribute, i.e., rating scale methods, category scaling, and magnitude estimation. As such methods may be thought of as yielding data which are relations on pairs of points from distinct sets (a point for each stimulus and a point for each response alternative) they satisfy the formal models used by data in Quadrants IIa and IIb.

SUMMARY

An abstract theory of psychological data has been constructed for the purpose of organizing and systematizing the domain of psychological methodology. It is asserted that from the point of view of psychological measurement theories all behavioral observations satisfy, at the simplest level, each of three dichotomies, generating eight classes called octants which were organized into four quadrants. Any behavioral observations when mapped into data involve accepting a miniature behavioral theory implicit in the method used to analyze the data. All of the various kinds of data were illustrated and some of the interrelations within and between quadrants were pointed out.

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A BRIEF GLOSSARY

Individuals = undefined.

Stimuli = undefined.

Behavior = any potentially observable relation among individuals and stimuli.

Raw Data = observed relations among real world objects, for example, pass-fail, preferential choice, yes-no, accept-reject, amplitude, latency, inconsistency, etc.

Data = raw data mapped into relations between points.

A Psychological Space = an abstract space in which lie points corresponding to objects in the real world (stimuli and/or individuals) and the relations among points reflect the observed relations among real world objects.

A Psychological Measurement Model = a set of assumptions from which is derived a calculus to construct a psychological space from the data matrix.

A Method of Collecting Data = the rules and the lore for arriving at raw data.

A Behavior Theory = the mapping from raw data into data.

APPENDIX

The following sets are given:

$$D = \{1,2,\cdots d \cdots r\}$$
$$H = \{1,2,\cdots h \cdots t\}$$
$$I = \{1,2,\cdots i \cdots m\}$$
$$j = \{1,2,\cdots j,k,l,\cdots n\}$$

Axiom 1. There exists distinct sets $K^{(d)}$, $d \in D$, where each $K^{(d)}$ is a segment of the real line.

Definition. Let $K = \{x \,|\, x = (x^{(1)}, x^{(2)}, \cdots x^{(d)}, \cdots x^{(r)}\}$ where $x^{(d)} \in K^{(d)}$, in which the elements x are vectors in r-dimensional space. Let, in addition,

$$C \subseteq K, \qquad C = \{c_i \,|\, i \in I\}$$
$$Q \subseteq K, \qquad Q = \{q_i \,|\, j \in J\}$$
$$A = C \times Q, \quad A = \{(c_i, q_i)\}$$
$$A_i \subseteq A, \qquad A_i = \{(c_i, q_i) \,|\, i \text{ fixed}\}$$
$$B = Q \times Q, \quad B = \{(q_i, q_k)\}$$

The sets A, A_i, and B are sets whose elements are pairs of points. We will also have a need for sets whose elements consist of pairs of such pairs of points, in particular we construct the sets:

$$A \times A, \quad A_i \times A_i, \quad B \times B$$

Axiom 2. There exists a function p on $K \times K$ into the real line, which satisfies the following conditions for a distance function:

$$|p(a,b)| = |p(b,a)|$$
$$p(a,b) = O \Longleftrightarrow a = b$$
$$|p(a,b)| \leq |p(a,c)| + |p(b,c)|$$

Axiom 3. Given two vectors differing only in one component, the sign of p is determined by that one component.

Axiom 4. Every measurement model satisfies the following three conditions:

1. A relation exists on a pair of points, or a pair of pairs of points.
2. The elements of the pair of points are drawn from two distinct sets as in A or from one set as in B.
3. The relation is either a proximity relation (O) or an order relation $(>)$.

Axiom 5. To each triple (h,i,j) and to each quadruple (hi,jk) corresponds a subset $D' = D'(h,i,j)$ or $D'' = D''(hi,jk)$ of D, that is, $D' \subseteq D$, $D'' \subseteq D$. D' or D'', as the case may be, will be called *the set of relevant dimensions.*

Definition.

1. q_{hij} is the projection of the vector q_j in the set of relevant dimensions, D' or D''.
2. c_{hij} is the projection of the vector c_i in the set of relevant dimensions, D'.
3. $p_{hij} = p(c_{hij}, q_{hij})$ is the image of the ordered pair (c_{hij}, q_{hij}), (the "distance" between the pair of points) in the set of relevant dimensions, D'.
4. $p_{hi,jk} = p(q_{hij}, q_{hik})$ is the image of the function p in the set of relevant dimensions D''.

Definitions of the information in the behavior mapped into the various quadrants (see Fig. 1).

Quadrant Ia

$$|p_{hij}| - |p_{hik}| \leq 0 \langle = \rangle j > k$$

where: $j > k$ signifies responses of the form "j preferred to k."

Quadrant Ib

$$||p_{hij}| - |p_{hik}|| \leq \epsilon_{hi,jk} \langle = \rangle j \dot{M} k$$

where: ϵ is a nonnegative number and $j M K$ signifies responses of the form "I cannot choose between j and k" or "I do not prefer one more than the other." (The symbol M is used to signify "*matches in preference*.")

Quadrant I with an intermediate category of responses

$$|p_{hij}| - |p_{hik}| < -\epsilon_{hi,jk} \langle = \rangle j \geq k$$
$$||p_{hij}| - |p_{hik}|| \leq \epsilon_{hi,jk} \langle = \rangle j \dot{M} k$$
$$|p_{hij}| - |p_{hik}| > \epsilon_{hi,jk} \langle = \rangle k > j$$

Quadrant IIa

$$p_{hij} \geq 0 \langle = \rangle i > j$$

where: $i > j$ signifies responses of the form the individual, i, passes, accepts, etc., the stimulus j. More generally, an element i of one set dominates an element j of another set.

Quadrant IIb

$$|p_{hij}| \leq \epsilon_{hij} \langle = \rangle i \dot{M} j$$

where: $i \dot{M} j$ signifies responses of the form the individual i says yes, agrees, endorses, etc., the stimulus j. More generally, an element i of one set is matched with an element j of another set.

Quadrant II with an intermediate category of responses

$$p_{hij} > \epsilon_{hij} \langle = \rangle i > j$$
$$|p_{hij}| \leq \epsilon_{hij} \langle = \rangle i \dot{M} j$$
$$p_{hij} < \epsilon_{hij} \langle = \rangle j > i$$

Quadrant IIIa

$$p_{hi,jk} \geq 0 \langle = \rangle j > k$$

where: $j > k$ signifies responses of the form "j is greater than k."

Quadrant IIIb

$$|p_{hi,jk}| \leq \epsilon_{hi,jk} \langle = \rangle j M k$$

where: $j M k$ signifies responses of the form "these stimuli j and k are not different, they match."

Quadrant III with an intermediate category of response

$$p_{hi,jk} > \epsilon_{hi,jk} \langle = \rangle j > k$$
$$|p_{hi,jk}| \leq \epsilon_{hi,jk} \langle = \rangle j M k$$
$$p_{hi,jk} < -\epsilon_{hi,jk} \langle = \rangle k > j$$

Quadrant IVa

$$|p_{hi,jk}| - |p_{hi,lm}| \leq 0 \langle = \rangle (j,k) < (l,m)$$

where: $(j,k) < (l,m)$ signifies responses of the form "the pair of stimuli (j,k) are more alike than the pair of stimuli (l,m)."

Quadrant IVb

$$||p_{hi,jk}| - |p_{hi,lm}|| \leq \epsilon_{hi,jk,lm} \langle = \rangle (j,k) M (l,m)$$

where: $(j,k) M (l,m)$ signifies responses of the form "the pair of stimuli (j,k) are no more different than are the pair of stimuli (l,m)."

$$|p_{hi,jk}| - |p_{hi,lm}| < -\epsilon_{hi,jk,lm} \langle = \rangle (j,k) < (l,m)$$
$$||p_{hi,jk}| - |p_{hi,lm}|| \leq \epsilon_{hi,jk,lm} \langle = \rangle (j,k) M (l,m)$$
$$|p_{hi,jk}| - |p_{hi,lm}| > \epsilon_{hi,jk,lm} \langle = \rangle (l,m) < (j,k)$$

DIRECTIONAL STATISTICAL DECISIONS

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This paper has two purposes. First, we shall point out a seemingly common logical error in the statistical interpretation given results of two-sided tests of statistical hypotheses. A correct interpretation of the traditional two-sided test would appear to make this classic procedure of essentially negligible interest. Second, we shall outline an appropriate treatment of the problem with which two-sided statistical tests seem concerned and contrast this procedure with the one-sided test. Throughout the paper, we shall indicate the relationship of our discussion to the prolonged controversy on one-sided tests versus two-sided tests (Burke, 1953, 1954; Goldfried, 1959; Hick, 1952; Jones, 1952, 1954; Kimmel, 1957; Marks, 1951, 1953).

The arguments developed in this paper are based on logical considerations in statistical inference. (We do not, of course, suggest that statistical inference is the only basis for scientific inference.) Our statistical interpretation and development stem primarily from the decision-theoretic position of Wald (1939, 1950).

THE NONDIRECTIONAL TWO-SIDED TEST

Consider the traditional two-sided test. For example, we wish to test the null hypothesis:

$$H_2: \mu_X - \mu_Y = 0$$

against the obvious two-sided alternative:

$$H_{13}: \mu_X - \mu_Y \neq 0$$

where μ_X and μ_Y are the population means of the normally distributed random variables X and Y, and where $\sigma_X = \sigma_Y = \sigma$ is unknown.[1] This, of course, is an example of the classic t test.

The error in statistical interpretation of perhaps the majority of those who have used this test lies in the decision or statement made if the null hypothesis is rejected. When this occurs obviously we accept the alternative hypothesis that the population means are different. While this is correct, *we cannot logically make a directional statistical decision or statement when the null hypothesis is rejected on the basis of the direction of the difference in the observed sample means.* Our a priori alternative hypothesis merely states a nondirectional difference; logically, then, we may only state or decide upon a nondirectional difference if this alternative is accepted.

It seems difficult to imagine a problem for which this traditional test could give results of interest. To find a difference or a "significant" effect and not be able to decide in which direction this difference or effect lies, seems a sterile way to do business. One escape would be to conduct the traditional nondirectional two-sided test, and then if the alternative hypothesis is accepted, to gather new data and attempt to decide upon the direction provoked by the original nondirectional two-sided affair with the appropriate

[1] Our designation of the null hypothesis as H_2 rather than H_0 is unconventional; however, the exposition seems logically clearer if we use the subscripts 1, 2, and 3 to refer to negative, zero, and positive differences, respectively.

one-sided test. This two-stage procedure, while correct, obviously wastes data. A more efficient, single-stage procedure is described in the section after next.

THE DIRECTIONAL ONE-SIDED TEST

Consider the one-sided test. We wish, for example, to test the null hypothesis:

$$H_{12}: \mu_X - \mu_Y \leq 0$$

against the one-sided alternative:

$$H_2: \mu_X - \mu_Y > 0$$

One point of confusion concerning the above statement of the null hypothesis sometimes occurs because traditionally the definition of a null hypothesis has been restricted to the hypothesis of no difference—e.g., our H_2 of the previous section. Under this latter interpretation, the one-sided test would be for deciding between the null hypothesis H_2 and the alternative H_3—leaving the left flank unguarded. Statistically, this restriction is not necessary; a statistical hypothesis simply is a statement about the probability distribution(s) of observable random variable(s) (Neyman, 1950, p. 250). Any such statement, such as our H_{12} above, if it is the hypothesis being tested (in the sense that falsely rejecting it may occur with maximum probability given by the level of significance) is then a null hypothesis (Neyman, 1950, p. 259). The well-entrenched adjective "null" is probably misleading for it implies an unnecessary restriction on statements of hypotheses to be tested.

On the other hand, Burke (1953, 1954) has argued not unconvincingly that stating the null hypothesis in a one-sided test as a nonpositive difference may often be scientifically naive; the difference between the scientific hypotheses corresponding to H_1 and H_2 may be such that it would not be wise (extrastatistically) to toss them into the same null pot, where they remain indistinguishable.

Of course, with the one-sided test we are in the much more palatable position than with the traditional two-sided test of being able to make a directional statistical decision if the alternative hypothesis is accepted.

THE DIRECTIONAL TWO-SIDED TEST

Let us say we are interested in making a directional decision if we attain "statistical significance" and yet wish to guard against differences in both directions. This section outlines a solution of this problem for the example considered here, statistical decisions about differences between population means of normally distributed random variables with equal variance.

To do this we consider briefly the notion of a statistical decision function (Wald, 1950). A statistical decision function prescribes a correspondence between one of k decisions as a function of n possible observational outcomes (Neyman, 1950, p. 10). In applied statistics n is usually infinite; for example, in our problem the possible values of t are the $n = \infty$ possible outcomes. On the other hand, in conventional applied statistics k is usually either two or infinite; when k is two, we have hypothesis testing using two-valued statistical decision functions, and when k is infinite we have the problem of estimation (deciding along a continuum of points or intervals). Either of the t tests considered in the two previous sections uses a two-valued statistical decision function or, less solemnly, is a two-decision procedure, because in each case there are two, and only two, possible decisions contemplated: a decision to accept (not

reject) the null hypothesis or a decision to accept the alternative. The two two-decision procedures are different, of course, both because of the nature of the hypotheses tested and because of the different correspondence established between the possible outcomes and the two decisions; i.e., the critical regions or tail(s) for the two tests are different for rejecting the null hypothesis.

Wald's (1939, 1950) contribution of the notion of a statistical decision function integrates into a single general theory what prior to 1939 were thought of as two more or less distinct branches of statistics, hypothesis testing and estimation. In this most general framework, conventional hypothesis testing is represented by two-valued statistical decision functions while estimation involves statistical decision functions of infinitely many values.

However, there is no reason why we should not consider the zone in-between: k-decision procedures, $2 < k < \infty$. And this is precisely what we shall do to give a correct single-stage solution to the directional two-sided decision problem. For this problem requires a *three*-valued statistical decision function (Lehmann, 1950); we wish to decide among

$$H_1: \mu_X - \mu_Y < 0$$

and

$$H_2: \mu_X - \mu_Y = 0$$

and

$$H_3: \mu_X - \mu_Y > 0$$

The difference between the traditional nondirectional two-sided test and the directional two-sided test proposed in this section may be seen by considering the possible errors which may occur in making a wrong decision. For the classic nondirectional test, only two errors are possible: (*a*) the error of deciding that

there is a difference, when, in fact, the null hypothesis is true—an error of the first kind (α error), or (*b*) the error of not detecting that the null hypothesis is false, i.e., deciding that there is no difference when in fact there is—an error of the second kind (β error). The four possible situations may be represented conveniently in the following four-fold table:

Nature

		H_2	H_{12}
Decision about Nature	H_2	correct decision	β error
	H_{12}	α error	correct decision

For the directional test of this section, there are six possible errors. They may readily be seen as the off-diagonal cells in the following nine-fold table:

Nature

		H_1	H_2	H_3
Decision about Nature	H_1	correct decision	α_{12} error	γ_{13} error
	H_2	β_{21} error	correct decision	β_{23} error
	H_3	γ_{31} error	α_{32} error	correct decision

Any one of the possible errors in the above table is symbolized uniquely by the subscripts used: the first subscript indicates which hypothesis is decided upon, while the second indicates the hypothesis which obtains in Nature. We have added the unnecessary α, β, and γ, to provide comparability with the notation used in classical hypothesis testing. Thus, the α_{12} and α_{32} errors are similar to the α error in the usual two-decision

FIG. 1. Performance characteristic of the nondirectional two-sided test with $\alpha = .05$. State of Nature in units of the standard error of the difference between the means. Rule of inductive behavior (for large samples): decide upon H_2 when $-1.960 \leq t \leq +1.960$, decide upon H_{13} when $|t| > +1.960$.

STATE OF NATURE
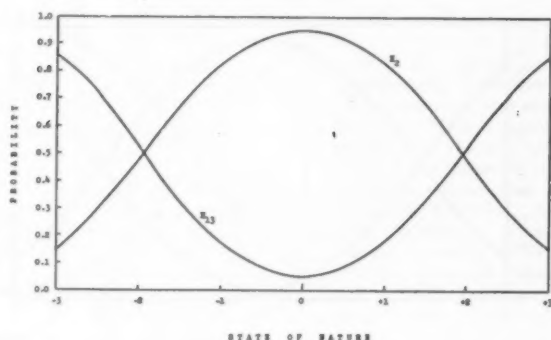
problem; either involves making a false decision of difference when there is none. The β_{21} and β_{23} errors are similar to the β error; either involves not detecting a difference. The particularly repugnant γ_{13} and γ_{31} errors —"errors of the third kind"—have no parallel in classical hypothesis testing, as these "gamma errors" involve deciding upon a difference in the wrong direction.[2]

The difference between the directional and nondirectional two-sided tests may be illustrated quantitatively if we contrast their *performance characteristics*. The performance characteristic of a k decision procedure is the system of k functions, each of which gives the probability, as a function of the model describing the state of Nature, of accepting one of the k decisions contemplated (Neyman, 1950, p. 11). Figure 1 shows the performance characteristic for the classic nondirectional two-sided (equal tails) t test with level of significance α ($= .05$). It consists of two functions, each giving the probability of deciding upon the hypothesis (H_2 or H_{13}) indicated.[3] Note that the two

[2] Mosteller (1948) seems to have coined the expression "errors of the third kind."

[3] Dixon and Massey (1957, Ch. 14) and Walker and Lev (1953, pp. 161–167) give excellent elementary discussions of how these functions may be computed.

functions are redundant; the curve giving the probability of accepting H_2, Wald's operating characteristic, is complementary to the curve giving the probability of accepting H_{13}, the Neyman-Pearson power function. More generally, of course, any performance characteristic has this sort of redundancy: since the k possible decisions are mutually exclusive and jointly exhaustive, the probability of making any $k - 1$ of them is sufficient to give the desired information.

Figure 2 shows the three functions of the performance characteristic of our three-decision procedure, the directional two-sided test. For this illustration the probability of making each of the α_{12} and α_{32} errors has been set at one-half the level of significance used for Fig. 1; this makes it convenient to compare the directional and nondirectional two-sided tests when the directional test is carried out under the guise of the traditional nondirectional test with level of significance α. Several comparisons of this three-decision procedure with the traditional test seem worth mentioning. When $\alpha_{12} = \alpha_{32} = \frac{1}{2}\alpha$, comparing Figs. 1 and 2:

1. The probability of accepting the null hypothesis H_2 is the same for either test for all states of Nature.
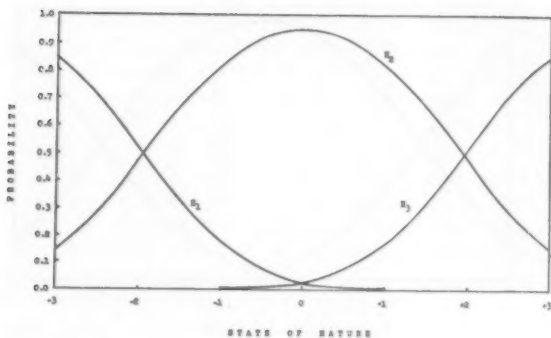
2. The probability of correctly ac-

FIG. 2. Performance characteristic of the directional two-sided test with $\alpha_{12} = \alpha_{32} = .025$. State of Nature in units of the standard error of the difference between the means. Rule of inductive behavior (for large samples): decide upon H_1 when $t < -1.960$, decide upon H_2 when $-1.960 \leq t \leq +1.960$, decide upon H_3 when $t > +1.960$.

cepting either H_1 or H_3 in the directional test is less than the probability of correctly accepting H_{13} in the nondirectional test, and, for a given state of Nature, this loss of power is equal to the probability of making the nasty error of the third kind.

3. The probability of making a gamma error is always less than $\frac{1}{2}\alpha$.

An alternative treatment for the directional two-sided problem would be to make the probabilities of each of the α_{12} and α_{32} errors traditional values, like .05 or .01, rather than the .025 or .005 generated by an incorrect interpretation of the nondirectional test.

DISCUSSION

It seems obvious that the traditional two-sided test should almost never be used. If, as is typical, not rejecting the null hypothesis is a result of little scientific concern, then this test may be said never to give results of direct scientific interest because accepting the nondirectional alternative H_{13} is merely a generator of directional alternative hypotheses.[4]

[4] When the alternative hypothesis is so nonspecific as the nondirectional H_{13}, a compelling argument (Jones, 1955) may be made not to test hypotheses at all—that a more appropriate statistical procedure is estimation, e.g., for our problem, first find a point esti-

Since we are proposing that almost without exception the directional two-sided test should replace the traditional nondirectional test, it seems appropriate to contrast the one-sided test with this three-decision procedure. The performance characteristic of the one-sided test is given in Fig. 3. The level of significance ($\alpha = .05$) in this illustration is the same as in Fig. 1. In comparing Figs. 2 and 3, then, we compare the one-sided test with the three-decision procedure where $\alpha_{12} = \alpha_{32} = \frac{1}{2}\alpha$. The loss of performance engendered by guarding both sides with the directional two-sided test is readily seen. Only from the traditional point of view of correctly accepting the null hypothesis, or controlling errors of the first kind, is the two-sided test as good; the two-sided test is markedly

mate for $\mu_X - \mu_Y$ and then determine a confidence interval about this point. Indeed, it might be argued that even the directional alternatives H_1 and H_3 are too nonspecific relative to the null hypothesis H_2, because their dimensionality in the parameter space is greater than that of H_2. A completely "balanced" or "symmetric" theory of testing hypotheses would seem logically to require that all k hypotheses under consideration have the same dimensionality in the parameter space. Such symmetry for our procedure could occur if H_1 and H_3 were chosen as specific negative and positive differences rather than any positive and negative differences.
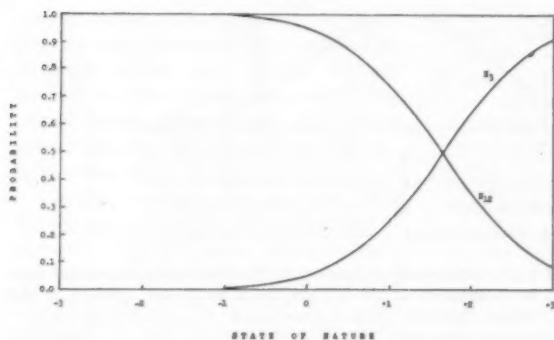
FIG. 3. Performance characteristic of the directional one-sided test with $\alpha = .05$. State of Nature in units of the standard error of the difference between the means. Rule of inductive behavior (for large samples): decide upon H_{12} when $t \leq +1.645$, decide upon H_3 when $t > +1.645$.

less powerful (in the Neyman-Pearson sense) than the one-sided test for correctly rejecting the null hypothesis in the "right" direction, from the viewpoint of the one-sided test. Also, with the two-sided test, there is always the possibility of the repulsive gamma errors.

To equate the power of the directional two-sided test with that of the one-sided test with level of significance α, it is sufficient to use this three-decision procedure with $\alpha_{12} = \alpha_{32} = \alpha$. Compare Figs. 3 and 4.

A nice feature of this comparison is that there is no difference in the critical values to t in the tail corresponding to the alternative hypothesis for the one-sided test. See Fig. 5. Thus the traditional and delicate problem of changing the number of

sides in midstream and/or fudging with a posteriori alpha values cannot arise. The distinction between these two tests lies in whether differences in the "wrong" direction, from the viewpoint of the one-sided test, can lead to a decision in this direction. For the three-decision procedure proposed in this paper, this may happen; for the traditional one-sided test, it may not, as the null hypothesis there includes all nonpositive differences. At first glance, then, it might seem that one would always prefer the three-decision procedure because it guards against differences in both directions —differences which of course may be decided upon directionally. It is suggested that this argument is not to be taken lightly; consult Burke (1953, 1954) for an extended and convincing

FIG. 4. Performance characteristic of the directional two-sided test with $\alpha_{12} = \alpha_{32} = .05$. State of Nature in units of the standard error of the difference between the means. Rule of inductive behavior (for large samples): decide upon H_1 when $t < -1.645$, decide upon H_2 when $-1.645 \leq t \leq +1.645$, decide upon H_3 when $t > +1.645$.
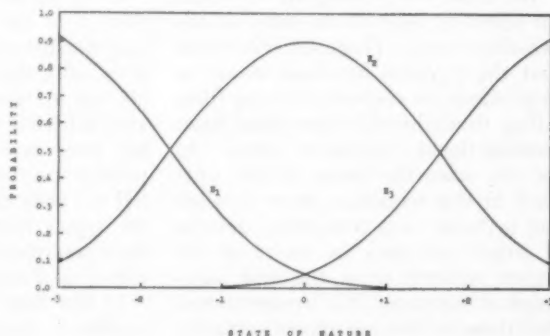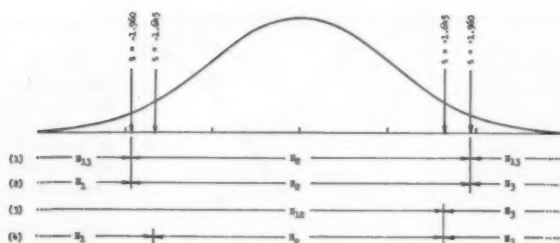
Fig. 5. Ranges of values of t (for large samples) leading to the decision indicated for four statistical tests. The numbers in parentheses at the left are the same as the numbers of the above figures giving the performance characteristics of these tests: (1) nondirectional two-sided test with $\alpha = .05$, (2) directional two-sided test with $\alpha_{12} = \alpha_{32} = .025$, (3) directional one-sided test with $\alpha = .05$, (4) directional two-sided test with $\alpha_{12} = \alpha_{32} = .05$.

defense of the scientific desirability of procedures which will detect differences in both directions.

However, the choice is not completely clear cut. Consider again Fig. 4, the performance characteristic of the three-decision procedure where $\alpha_{12} = \alpha_{32} = \alpha$. In comparing Figs. 3 and 4, one traditionally serious disadvantage of the directional two-sided test obtains: in the unlikely event that the null hypothesis H_2 is true, i.e., the population mean difference is exactly equal to zero, then the probability of accepting this null hypothesis is only $1 - \alpha_{12} - \alpha_{32} = 1 - 2\alpha$ for the two-sided test as compared with probability $1 - \alpha$ of accepting H_{12} when the one-sided test is used.

We have not attempted to settle the scientific issue of one-sided versus two-sided tests. However, it is hoped that the problem has been recast so as to eliminate confusion arising from failing to distinguish directional from nondirectional two-sided tests. As for the scientific issues briefly outlined in this section, a more detailed and perhaps more compelling defense of either test may be found in the papers referred to in the first paragraph of this paper, if it is remembered that these writers are almost certainly

referring to our three-decision procedure when speaking of two-sided tests.

Notes

The directional two-sided test proposed in this paper need not necessarily be developed explicitly as a three-decision procedure. An alternative approach would be simultaneously to make two one-sided tests (Hodges & Lehmann, 1954): H_{12} against H_3 and H_{23} against H_1. If both these two-decision procedures are carried out simultaneously and with the same data, at level of significance α, it is readily seen that we have exactly the equivalent of the three-decision procedure illustrated and described in Fig. 4.

It is perhaps worth pointing out that it is surely not necessary and may not always be best to set $\alpha_{12} = \alpha_{32}$ in our directional two-sided test; after all, the α_{12} and α_{32} errors may entail very different consequences. Indeed, we may envision a continuum of possible partitions of $\alpha_{12} + \alpha_{32}$ from a left tail critical one-sided test through the equal tails three-decision procedure described in this paper to a right tail critical one-sided test.

It has been convenient to discuss problems about differences between

means. The rationale and application of the three-decision procedure outlined above may easily be extended to other problems involving other parameters where the traditional alternative hypothesis lies on both sides of the null hypothesis.

Finally, it might also be noted that the statistical notion of the number of sides or "tails" bears no necessary relation to the scientific notion of whether the test is directional or nondirectional. For example, the one-sided t test is directional scientifically and one-sided statistically, while most traditional F and chi square tests are nondirectional scientifically and one-sided statistically. As such, directional decisions cannot properly be made with such F and chi square tests and they are to be thought of merely as hypothesis generators for scientifically more explicit statistical decision procedures.

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A THEORY OF DEPRESSION AND ENHANCEMENT IN THE BRIGHTNESS RESPONSE [1]

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In visual perception there are two phenomena which will be the concern of this paper. These are depression and enhancement, and we shall consider them as they relate to the brightness response of a subject (*S*).

The brightness of a test circle, for example, can be decreased (depressed) or increased (enhanced) if the circle is surrounded by an inducing field of greater or less luminance respectively (Heinemann, 1955). These things happen even though the luminance of the test field remains constant. If, on the other hand, we eliminate the surrounding field and, under various conditions, present the test field by itself, only enhancement is clearly evident; for example, if we measure the threshold luminance necessary to just see the test circle, and if the area of the circle is increased, threshold luminance decreases (Graham, Brown, & Mote, 1939) as shown in Fig. 20.

PHYSIOLOGICAL EXPLANATION

One plausible explanation for these effects is that they result from the interaction between the individual elements in the retina; e.g., between rods or cones, between bipolars, or between ganglion cells.

Inhibition

Actually, Hartline and Ratliff (1957) offer evidence that this is in fact the case with respect to depression. They show that in the eye of *Limulus*, the horseshoe crab, if the frequency of electrical discharge is recorded from a single optic "test" fiber leading from a single receptor, or ommatidium, this frequency will decrease, or be inhibited, if a neighboring "inducing" receptor is stimulated simultaneously. On the other hand, if the inducing fiber is itself somehow inhibited, the test fiber is disinhibited; that is, test frequency increases again. Finally, if one cuts the neural connections between the adjacent fibers, inhibition does not take place. It would seem, then, that psychological depression might be the result of a lateral inhibition, possibly at a test synapse, the amount of lateral inhibition being directly proportional to the frequency of the inducing fiber.

With respect to psychophysical enhancement, it has historically been explained by the physiological phenomenon of summation (Graham, Brown, & Mote, 1939). Actually, what has been called summation has been shown to take place in the vertebrate (frog) eye by Hartline (1940). Recordings were taken from a single optic fiber leading from a test receptor located at the approximate center of a field of stimulation on the retina. As the area of stimulation was increased, the frequency recorded from the test fiber increased. Other investigators have obtained similar results (Adrian & Matthews, 1928).

One possible explanation of this is that the inducing fibers, surrounding the test fiber, excited lateral neurons which in turn introduced summative rather than inhibitive impulses at the test synapse. Unfortunately, such an explanation requires the same mechanism, the synapse, to handle both inhibition and summation; either that, or two different physiological structures, such as two different types of synapses, must be hypothesized. Actually, an explanation favored by the present author is one that involves only the phenomenon of inhibition; i.e., what is thought to be summation is probably a form of disinhibition such as found by Hartline and Ratliff (1957).

In support of this statement we must first note that the experiments which were performed on Limulus never found summation. This is not only true of the experiment of Hartline and Ratliff (1957) but of Graham (1932) and also of Hartline, Wagner, and Ratliff (1956). Second to consider is that correlated with the absence of physiological summation in *Limulus* is the absence of the off fiber response in its retina (Granit, 1955, p. 33). Off fibers are found in the vertebrate eye; they are inhibited by light and discharge at the cessation of light (Granit, 1955, pp. 33–35).

Before we make something of this information, however, let us look at experiments where we do find summation. Hartline's investigation of the frog eye (1940) and Adrian and Matthew's of the eel eye (1928) are two such experiments (already noted above). In both of these experiments we find summation, but in addition we now find an abundance of rhythmic spontaneous discharges which occur along with the on discharges that are measured as the criterion responses. Such rhythmic spontaneous discharges have also been found in the cat eye (Barlow, FitzHugh, and Kuffer, 1954).

How can we organize all these facts into our disinhibition explanation of the above studies showing summation? Let us first assume that the spontaneous discharge comes from off fibers, and secondly, that surrounding the particular test areas of Hartline and Adrian and Matthews were many such spontaneously active off fibers. We would expect then that these active off fibers were inhibiting the on responses of the test receptor at the center of the test area; Granit describes in detail demonstrations of such "'on-off' antagonism" (Granit, 1955, pp. 69–72).

Our disinhibition explanation, therefore, is this: As the illuminated test area in the above studies was increased in size, it turned off more and more off fibers surrounding the test receptor through inhibition by light. This inhibition of off fibers occurs not only from direct light within the test borders, but also from the light scattered outside its borders (see Boynton, 1951). As more and more spontaneous off fibers were thus inhibited by the growing test area (Hartline, 1940; Adrian & Matthews, 1928), the central test receptor became disinhibited or, as found in the foregoing studies, the frequency of test fiber discharge increased. This then is the explanation favored by the present author for the Hartline and Adrian and Matthews summation.

One might ask at this point why stimulation of the on receptors that were also in Hartline's (1940) surround did not inhibit the test receptors, since it has been shown that on receptors can inhibit one another. Actually, as Hartline increased the luminance of the test field, the sum-

mative effect of increasing test area began to disappear, until at high luminances an increase in test area brought about a slight decrease in test receptor frequency; i.e., inhibition rather than summation. Thus, according to our above hypothesis, as on activity, from high test luminance, increased relative to off spontaneous activity, the increase in on inhibition was greater than the release from off inhibition, or the reverse of summation occurred as would be predicted by the above hypothesis.

We see then that a possible basic explanation exists to explain both physiological inhibition and summation in terms of inhibition alone. For inhibition, there is a blocking of the test impulse, possibly at a test pathway synapse, by impulses from lateral neurons excited by inducing photoelements or fibers. For summation, there is a release of inhibition, or a disinhibition, as the spontaneous off fiber activity in the test receptor surround, which normally inhibits the test fibers through lateral neurons, is itself inhibited.

If we now return to psychophysic l depression and enhancement, the next step is obviously to attempt an explanation of these phenomena in terms of physiological inhibition. Suppose we do this now in mathematical terms such that the psychophysical consequences of our physiological theory are derivable and testable.

Assumptions and Basic Formulations

Psychophysical Fields

The general plan of our theory is first to assume that any psychophysical visual field can be thought of in terms of its resulting visual retinal image and the number of retinal elements (i.e., rods or cones, or nerve fibers leading from them) included within the image borders. Psychophysical interaction between fields would therefore be the result of physiological inhibition between individual retinal elements. This means that in addition to the possible effects between fields, effects can take place within fields as well.

Four fields will be involved at one time or another (see Figs. 1, 5, 7, 9, 11, 13, 15, 17, 19): (a) the test field (t), on which an S makes a brightness discrimination; (b) the inducing field (i), the presence of which can affect test brightness; (c) the spontaneously active field (D), which can also affect test brightness; since its activity is assumed to be from off retinal elements, we might expect that direct or scattered light from the test and inducing fields will reduce such activity. The spontaneous field is also assumed to extend through the test and inducing field so that, for example, at very low test or inducing luminances some off fibers are still active within these fields; and (d) the match field (m), which is a psychophysical standard and to which the test field is subjectively matched in brightness. Different experiments employed fields of different shapes and sizes. In most of the experiments to be reported, all fields were confined to the fovea.

When a match field was used in order to measure suprathreshold test brightness, it was presented to the eye opposite to that viewing the test and inducing fields. One of two measures of test brightness was employed: match luminance or test luminance. For the first, if test brightness changed, match luminance was adjusted so that match brightness now equalled test brightness. For the second measure, if test brightness changed, test luminance was adjusted

so that test brightness could be brought back to equality with a constant match brightness. Both measures, of course, are to be considered in our formulation.

One final comment with respect to psychophysical fields is that it will become convenient later to talk about single-field experiments as opposed to multiple-field experiments. By our own definition, a single-field experiment is one in which a single illuminated field, the test field, is presented to the eye. In the multiple-field experiment, one or more additional fields (inducing fields) are presented besides the test field. In both single- and multiple-field experiments a match field may or may not be presented to the match eye.

Assumptions

At this time it might be best to give the reader a short overall view of our basic assumptions. In Fig. 1 are diagrammed the main or primary physiological events that we believe are basic to psychophysical depression and enhancement.

For depression of test brightness, the test field on fibers (which presumably mediate psychophysical brightness) are inhibited by (a) the inducing on fibers, (b) other test on fibers, and (c) the spontaneously discharging off fibers.

For enhancement of test brightness we must first note something not indicated in Fig. 1; i.e., off fibers exist within the test and inducing fields as well as the discharge field. Now to explain enhancement in a multiple-field situation; i.e., one in which an inducing field surrounds the test field, we must assume that when the inducing-field luminance is zero, the off fibers immediately surrounding the test field are normally active. As the inducing field now increases
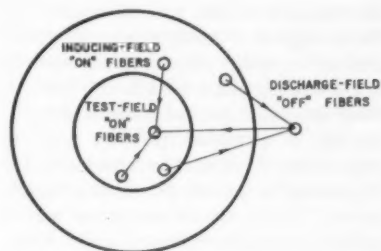

FIG. 1. Diagram of inhibitory interactions between test and inducing on fibers and spontaneously discharging off fibers. Arrows show directions of main inhibitory effects. Circles represent any fiber or fibers in each respective field. Additional possible interaction, e.g. discharge to inducing, test to inducing, etc. are considered to be of secondary importance, as will be discussed later.

from zero to some value below test-field luminance the off fiber activity within the inducing-field borders becomes diminished. Enhancement in the multiple-field situation is therefore explained as a "disinhibition" of test-field activity; i.e., inhibition by the inducing field of discharge activity releases the test field from discharge inhibition.

Enhancement also occurs quite strongly in single-field experiments in which the area of a single field is increased at a brightness close to threshold. Here our explanation assumes that as the test-field area increases, its spreading light turns off more and more off fibers, thus reducing the amount of off discharge inhibition which can affect test brightness. In other words, as test area increases it figuratively pushes back the discharge-field borders. For the single-field as well as the multiple-field situation, enhancement is explained in terms of disinhibition.

Since we are trying to explain psychophysical events in terms of underlying physiological events there are a number of different types of

assumptions which will be made: (*a*) Physiological assumptions. We will assume certain physiological events to be taking place within the human observer while he is behaviorally observing a stimulus pattern and is responding to what he observes. (*b*) Physiological-psychophysical assumptions. These are assumptions which will deal with the manner in which the physiological events are related to psychophysical behavior. (*c*) Psychophysical assumptions. We will assume that certain instructional stimuli always elicit certain behavior (i.e., brightness-matching behavior).

PHYSIOLOGICAL ASSUMPTIONS

Our physiological assumptions imply that certain physiological events are taking place in the human *S* as he behaves psychophysically. Actually, these assumptions are based upon physiological data taken from animals whose eyes are quite similar in many functional respects to human eyes.

Our physiological assumptions will involve both elemental and field concepts. The elemental concepts will have to do with individual retinal elements and the relations between these elements. Field concepts will

FIG. 2. Impulse frequency of a Limulus fiber as a function of the luminance stimulating the ommatidium. The line drawn through the data is according to Formula 1 in the text. (Data taken from Hartline & Graham, 1932.)

describe groups of such elements taken together.

Elemental Concepts

On activity. The assumption of on activity is basic to the psychophysical brightness response. Let us first deal with the physiological aspects of this assumption, however. We will assume that in the human retina there are individual retinal elements, each made up of a photoreceptor and attached fiber, called ɔn elements which are activated when light strikes the photoreceptor. Such on elements have been found in the eyes of many different animals such as the cat, pigeon, frog, guinea pig, etc. (Granit, 1955, pp. 63–78). Further, we shall assume that the frequency of response of these on fibers is directly proportional to the luminance of light striking the photoreceptor, or

$$f_t = k_1 B_t^{\frac{1}{4}} \qquad [1]$$

where f_t is the frequency of the stimulated fiber which we shall call the test fiber; B_t is the luminance of the stimulating light and k_1 is a proportionality constant. The proportionality constant stands for unspecified events which are important in transforming luminance into frequency. Proportionality constants will be used in this same manner for the remaining assumptions below.

The one-fourth power exponent of B_t in Equation 1 is based upon an empirical fit, presented in Fig. 2, to the data of Hartline and Graham (1932) who measured frequencies of single fibers in *Limulus* as a function of the intensity of the stimulating light. The data in Fig. 2 were taken after the stimulating light was on 3.5 sec. These 3.5-sec. data are important for our consideration since the behavioral data which our theory
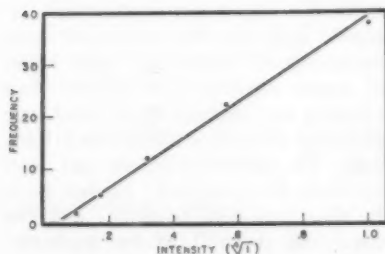
will eventually attempt to explain is concerned with maintained rather than instantaneous light stimulation. The fit in Fig. 2 seems satisfactory.[2] A similar fit can be made to data obtained in an experiment on the frog by Hartline (1938). He also measured frequency as a function of stimulating intensity and took his data 4 sec. after the stimulation light was turned on.

Off spontaneous activity. Our next physiological elemental assumption is that in the human retina there are individual retinal elements called off elements which are spontaneously active in the absence of light and are turned off when light strikes the retina. Off fibers have been found by Adrian and Matthews (1928) in the eel eye and by Hartline (1938; 1940) in the frog eye. In addition, in both of these investigations spontaneous electrical discharge was found in the nonstimulated retina. It should be pointed out, however, that their experiments did not attempt to show that the spontaneous discharge was in fact due to the firing of the off fibers. However, since off fibers were present in the retinae examined by the above two investigations, it seems reasonable to assume that the spontaneous discharge was due to off firing in these experiments.

A further assumption we would make is that frequency of the spontaneous off firing is proportional to the amount of some driving force. This force can be identified with the reticular activating system (although such identification is not crucial at this time). That is, according to Granit (1955, pp. 106–107) the reticu-

lar activating system, which can be considered to be a drive system, sends efferent fibers to the retina. Further, as described by Granit, stimulation of the reticular formation increases both on and off fiber activity. The assumption seems reasonable then, that the frequency of an off fiber in the retina is proportional to a driving source such as the reticular activating system. Therefore

$$f_d = k_2 D \qquad [2]$$

where f_d equals the frequency of a spontaneously discharging off fiber and D equals the amount of the driving force.

We are now ready to discuss our basic physiological assumption with respect to inhibition; i.e., when a fiber is inhibited its frequency will be reduced. Or

$$f_t = \frac{k_3}{h} \qquad [3]$$

where f_t equals the frequency of a test fiber and h is the inhibitory effect generated by another fiber. There are two types of such inhibition to be discussed. The first involves inhibition of the on fibers by other on and also by off fibers; and the second, involves inhibition of the off fibers by on fibers and also by direct or scattered light.

Inhibition of on fibers. Hartline and Ratliff (1957) have shown that in Limulus, the frequency of an on test fiber can be reduced if a nearby on inducing fiber is excited. They have shown further that the amount of reduction, or inhibition, is directly proportional to the frequency of the inducing fiber (see Fig. 3). Our assumption with respect to inhibition of on by on fibers, therefore, will be that inhibition (as indicated by test-fiber frequency reduction) is directly proportional to inducing-fiber fre-

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[2] The fit in Fig. 2 is with intensity (meter candles) rather than luminance (millilamberts) plotted along the abscissa. Actually, if luminance were to be plotted, the same straight line function would hold, as is described in Formula 1.
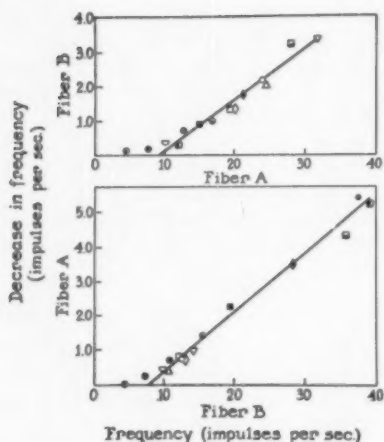
FIG. 3. Graphs showing mutual inhibition of two receptor units in the lateral eye of *Limulus*. In each graph the magnitude of the inhibition of one of the ommatidia is plotted (ordinate) as a function of the degree of concurrent activity of the other (abscissa). (From Hartline & Ratliff, 1957.)

quency and therefore, according to Equation 1, directly proportional to the luminance striking the photoreceptor attached to the inducing fiber, or

$$h_{i,t} = k_4 B_i^{\frac{1}{2}} \qquad [4]$$

where $h_{i,t}$ is the amount of inhibition produced by a single inducing fiber on a single test fiber, and B_i is the luminance of the light striking the inducing photoreceptor. As can be surmised, a double subscript indicates the effect of one element upon another, the inducing upon the test in this case.

There is one further assumption that must be made with respect to the inhibition of an on by an on fiber. This has to do with the spatial separation between the inducing and test fibers. Hartline, Wagner, and Ratliff (1956) show that inhibition becomes weaker with increased separa-

tion between an affected receptor and the region of the eye used to inhibit it. Therefore, we will assume that the inhibition of the test fiber is inversely proportional to the separation between a test and inducing fiber. Or

$$h_{i,t} = \frac{k_5}{r_{i,t}} \qquad [5]$$

where $r_{i,t}$ is the spatial separation between the inducing and test fibers.

With respect now to inhibition of on test fibers by off fibers in the spontaneous discharge field, we will assume that the off fiber, which is spontaneously discharging, affects an on fiber in the same way that another on inducing fiber might affect it. The fact that an off fiber can inhibit an on fiber is supported by data reported by Granit (1955, pp. 69–72) which points to an antagonistic action between on and off fibers, the excitation of one reducing frequency of another. Thus

$$h_{d,t} = \frac{k_5 D}{r_{d,t}} \qquad [6]$$

where $h_{d,t}$ is the inhibiting effect of the spontaneous off fiber on the test fiber, D is the driving force referred to above (which is somewhat comparable to luminance in that it determines the frequency of the off fiber), and $r_{d,t}$ is the separation between an off fiber and a test on fiber.

This, then, completes our assumptions with respect to the inhibition of on fiber activity. Let us now turn our attention to the inhibition of off fiber activity.

Inhibition of off fibers. As was found by Hartline (1938), an off fiber is inhibited when a light stimulus strikes the retina. Since, in our experimental situations, the beamed light, which becomes focused into the retinal test and inducing fields, produces both direct and scattered light

on the retina in effective amounts (Boynton, 1951), the relationship between the inhibition of off fiber frequency and the luminance of light directed toward the retinal test and inducing fields is not a simple one; both the direct off mechanism and the scattering mechanism enter into this relationship. In addition we must consider that the on fibers themselves contribute to inhibition of off fibers according to Granit's findings of mutual on-off antagonism (Granit, 1955, pp. 69–72). Therefore, for the present let us assume merely that inhibition of an off fiber is some function (a) of the luminance of the light directed toward either a test or an inducing on fiber, and (b) of the separation between the off and the test and inducing fibers. Or

$$o_{t,d} = f(B_t, r_{t,d}) \qquad [7]$$

$$o_{i,d} = f(B_i, r_{i,d}) \qquad [8]$$

where $o_{t,d}$ and $o_{i,d}$ represent inhibition respectively of an off fiber by (a) the test-field direct or scattered light or by a test on fiber; and (b) by inducing-field direct or scattered light or by an inducing on fiber. The terms $r_{t,d}$ and $r_{i,d}$ refer respectively to the spatial separation between (a) the test on and discharge off fibers, and (b) inducing on and discharge off fibers.

Physiological Field Concepts

At this point we have completed a summary of our assumptions concerning the elemental physiological relationships; i.e., between the on test, the on inducing and the off discharge individual fibers. It is obvious, however, that in the normal human, these fibers are excited in groups. That is, we can talk about a retinal test field which includes many active on test fibers as well as relatively inactive off

spontaneous fibers. (These are relatively inactive because of the focused light striking within the test field.) Also, we can talk about a retinal inducing field which includes many active on inducing fibers as well as relatively inactive off spontaneous fibers. Finally, we can talk about a discharge field which contains active off fibers and relatively inactive on fibers. Fig. 1 illustrates the locations of these fibers.

Field area-separation inhibitory effect. A special problem arises in describing the effect of one retinal area upon another if we try to do this in terms of the individual effects of single elements in one area upon single elements in another area. This problem arises in our situation particularly because of the fact that the separation between elements is a parameter of the response we are measuring. That is, we would want to describe the spatial separation between our two fields in terms of the separations between the elements in the two fields. However, if we take as our field separation some average separation $\bar{r}$ of all the separations between the elements of the two fields, we can see that this average separation $\bar{r}$ will not only be a function of the individual r values but also a function of the areas of the two fields being considered. That is, any change in the area of either field will introduce elements at different points in space and therefore at different r distances from other elements. As developed in detail by Diamond (1955), an accurate description of this situation would require a summation of all the interactions between individual elements as follows:

$$C_{i,t} = \Sigma \left(\frac{\Delta a_i}{r_{i,t}} \right) \qquad [9]$$

where Δa_i is the area of a single element in the inducing field and $C_{i,t}$ is

therefore a term specifying the combined effect of area and separations between individual elements in the test and inducing fields. A more convenient approximation of Formula 9 is:

$$C = k_7 \frac{.16E}{\bar{r}} \qquad [10]$$

where E equals the total number of inducing elements and .16 is the retinal area in square minutes taken up by a foveal cone. Since Formula 10 applies to any field or fields, no subscripts are used. The retinal area covered by a foveal element is computed according to Polyak's value of .4 min. for the average diameter of a foveal cone (Polyak, 1941). In a retinal cross section, therefore, each cone should account for a square (rather than a round) area of .16 min. (see Fn. 2). The value $\bar{r}$ between any two fields is calculated as the summation (or an appropriate approximation thereof) of all the r values between individual elements in the two fields divided by the total number of element combinations yielding r values.

From Formulae 5 and 9 therefore, we can see that group inhibition is a function of our area-separation term, or

$$H_{i,t} = k_8 C_{i,t} \qquad [11]$$

where H equals group inhibition from an inducing to a test field, and $C_{i,t}$ is the area-separation term involving these two fields.

Field luminance and the inhibition of on by on fibers. With respect to the relation of inhibition by on fibers to the luminance of light exciting them, it is relatively simple to derive the field assumptions from the individual elemental assumptions. That is, since, as the area of a field changes, there is no change in the average luminance affecting all the elements within the field, the group inhibition,

therefore, between two fields is directly proportional to the luminance of the inhibiting field, or

$$H_{i,t} = k_9 B_i^1 \qquad [12]$$

where B_i is the luminance of the inhibiting field.

Field inhibition of on by off fibers. Here again the step from the individual elemental analysis to the field analysis is a direct one. That is, each individual off fiber inhibits each individual on test fiber proportionally to D, the force driving the off fiber (see Formula 6), and to the area of the discharge field; therefore

$$H_{d,t} = K_{10} D k_{11} C_{d,t} \qquad [13]$$

where $H_{d,t}$ is the field inhibition from the discharge field to the test field, $C_{d,t}$ describes the area of the discharge field at the average separation between discharge elements and elements of the test field (see Formula 10).

Field inhibition of off fibers by direct or scattered light or by on fibers. Again we can proceed directly from the elemental relations to the group relationship; thus from Formulae 7 and 8

$$O_{t,d} = f(B_t, C_{t,d}) \qquad [14]$$

$$O_{i,d} = f(B_i, C_{i,d}) \qquad [15]$$

where $O_{t,d}$ and $O_{i,d}$ represent inhibition of the off fibers in the discharge field by (a) direct and scattered light and by (b) on fibers in the test field and in the inducing field respectively. $C_{t,d}$ and $C_{i,d}$ represent the area-separation relations between the test and discharge fields and the inducing and discharge fields respectively.

If we now return to assess the discharge field effect upon our on test fibers (see Formula 13) we find that it has been diminished by O inhibition, or

$$H_{d,t} = \frac{k_{10}Dk_{11}C_{d,t}}{O_{t,d} + O_{i,d}} \quad [16]$$

Summary of different possibilities of field inhibition. There are a number of specific interactions within or between fields which we shall now summarize and also give terms to. Before listing these interactions, however, we should distinguish between what we will call primary and secondary, or backward, inhibition (see Hartline & Ratliff, 1957).

A primary inhibitory effect is that of the inducing element upon the test element. A backward one is of the test upon the inducing element. Let us first list and specify in detail what the primary H and O inhibitory effects are and then consider the backward ones. Primary field effects are as follows: (a) within test inhibition:

$$H_{t,t} = K_1 B_t^{\frac{1}{2}} K_2 C_{t,t} \quad [17]$$

The double subscripts here indicate the effect of "inducing" test elements upon other test elements, or within-field inhibition. The luminance term requires only one subscript, since it concerns only the inducing element of a pair.

(b) within match:

$$H_{m,m} = K_1 B_m^{\frac{1}{2}} K_2 C_{m,m} \quad [18]$$

(c) inducing on test:

$$H_{i,t} = K_3 B_t^{\frac{1}{2}} K_4 C_{i,t} \quad [19]$$

(d) discharge on test:

$$H_{d,t} = \frac{K_5 D K_6 C_{d,t}}{O_{t,d} + O_{i,d}} \quad [20]$$

(e) discharge on match:

$$H_{d,m} = \frac{K_5 D K_6 C_{d,m}}{O_{m,d}} \quad [21]$$

(f) inducing on discharge (off inhibition):

It should be noted that $O_{i,d}$ (inducing on discharge inhibition) is an inverse function of the average separation, $\bar{r}_{i,t}$, between test and inducing fields. That is, the effectiveness of $O_{i,d}$ is greater when an inducing square, for example, is near the test square. This is because discharge elements near the test field are less effective than those far away. When the inducing field inhibits near discharge elements, the effect of spontaneous discharge on the test field is much more reduced than when the far discharge elements are inhibited. Therefore

$$O_{i,d} = f\left(\frac{1}{\bar{r}_{i,t}}\right) \quad [22]$$

and combining Formulae 15 and 22:

$$O_{i,d} = f\left(\frac{B_{i,i} C_{i,d}}{\bar{r}_{i,t}}\right) \quad [23]$$

(g) test on discharge:

$$O_{t,d} = f(B_t C_{t,d}) \quad [24]$$

(h) match on discharge:

$$O_{m,d} = f(B_m C_{m,d}) \quad [25]$$

We would now want to put together all of these assumptions into a combined formula which might predict the average frequency of the fibers in the test field for the multiple-field situation and also for the single-field situation. Thus, according to Equations 1, 3 and 17–25 for the multiple field:

$$F_t = \frac{k_1 B_t^{\frac{1}{2}}}{H_{t,t} + H_{i,t} + H_{d,t}} \quad [26]$$

where F_t equals the average frequency of the fibers in the test field, $H_{t,t}$ represents inhibition from within the test field, $H_{i,t}$ represents inhibition from the inducing field, and $H_{d,t}$ represents inhibition from the discharge field. And for the single-field situation:

$$F_t = \frac{k_1 B_t^{\frac{1}{2}}}{H_{t,t} + H_{d,t}} \quad [27]$$

Since the match-field pattern represents a single-field situation then:

$$F_m = \frac{k_1 B_m^{\frac{1}{2}}}{H_{m,m} + H_{d,m}} \qquad [28]$$

PHYSIOLOGICAL-PSYCHOPHYSICAL ASSUMPTIONS

Having finished with the physiological elemental and field assumptions, we will now go on to those assumptions involving relationships between the physiological events assumed to be taking place in the eye, and those psychophysical events measured externally.

The first such relationship about which we will make an assumption is between measured luminances of the various external psychophysical fields and the luminance of light that strikes the retinal elements. This relationship will depend upon the size of the pupil of the eye, light absorption and scattering in the interocular media, etc. (Bartley, 1951). Our assumption with respect to this relationship is that physiological luminance is directly proportional to psychophysical
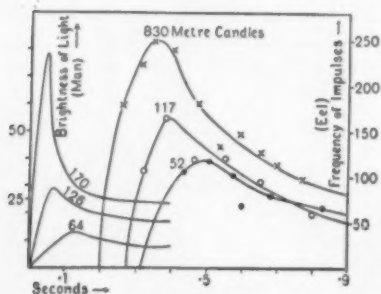

FIG. 4. Comparison of the increase in brightness, measured psychophysically in humans, with the increase in nerve impulse frequency, measured physiologically from the optic nerve of the eel. (After Adrian, 1928.)

luminance or

$$B' = k_{12}B \qquad [29]$$

where B' represents physiological luminance, B represents psychophysical or retinal luminance, and k_{12} is a proportionality constant representing the combined effects of the pupil, the interocular media, and such other effects that might determine the above relationship.

Similarly we must consider that our physiological area term, C_2 can be related to a psychophysical area term C_1. Actually, since our psychophysical measure of C_1 is in terms of visual angle C_1 can be assumed to exactly equal C_2.

The next physiological-psychophysical assumption to be made is that psychophysical brightness is directly proportional to the frequency of nerve discharge in the optic nerve. That is

$$A_t = k_{13}F_t \qquad [30]$$

where A_t stands for brightness of a test field, F_t is the average frequency of the optic nerve fibers stimulated within and leading from the test-field image on the retina, and k_{13} is a proportionality constant. There is no direct evidence to support this assumption although indirect support can be seen in Adrian (1928). Adrian compared the increase in the sensation of brightness, measured psychophysically in humans by Broca and Sulzer (1902), with the increase in frequency of the nerve impulse in the optic nerve of the eel; the duration of the stimulating light was varied in both cases. The shapes of the functions, brightness or frequency versus duration of light stimulation, are surprisingly the same (see Fig. 4). In other words although this does not constitute direct proof, this is what

we would expect if brightness were dependent upon the frequency of optic nerve discharge.

If we assume now, as in Formula 30, that brightness is proportional to frequency, it follows then according to Formulae 17–21 and 26–30, for the multiple-field situation

$$A_t = \frac{k_{12}k_{13}B_t^{\frac{1}{4}}}{K_1B_t^{\frac{1}{4}}K_2C_{t,t} + K_2B_t^{\frac{1}{4}}K_4C_{i,t} + \dfrac{K_5DK_6C_{d,t}}{O_{t,d} + O_{i,d}}} \quad [31]$$

or the brightness of the test field is directly proportional to its physiological luminance and inversely proportional to intratest, inducing and discharge physiological inhibition. And for the single-field situation

$$A_t = \frac{k_{12}k_{13}B_t^{\frac{1}{4}}}{K_1B_t^{\frac{1}{4}}K_2C_{t,t} + \dfrac{K_5DK_6C_{d,t}}{O_{t,d}}} \quad [32]$$

where the brightness of the test field is directly proportional to its luminance and indirectly to its within inhibition as well as inhibition from the discharge field.

Also the brightness of the match field A_m would be treated as a single-field situation, or

$$A_m = \frac{k_{12}k_{13}B_m^{\frac{1}{4}}}{K_1B_mK_2C_{m,m} + \dfrac{K_5DK_6C_{d,m}}{O_{m,d}}} \quad [33]$$

PSYCHOPHYSICAL ASSUMPTIONS

There is only one purely psychophysical assumption, which nevertheless must be made explicit. This involves the fact that in most of the experiments our theory will consider, the S's are required to match a test field and a match field in brightness. Our assumption is, therefore, that our instructional stimuli to S actually do effect this situation such that

$$A_t = A_m \quad [34]$$

where A_t equals test-field brightness and A_m equals match-field brightness.

Finally, treating our B and C terms as psychophysical terms, and setting equal the equivalents of A_t and A_m, (see Formulae 31 and 33) we see that for a multiple-field situation, in which there is an inducing field present

$$\frac{k_{12}k_{13}B_m^{a/4}}{K_1B_m^{b1/4}(K_2C_{m,m})^{c_1} + \dfrac{K_5D^{b_4/4}(K_6C_{d,m})^{c_2}}{K_7B_mC^{e}_{m,d}}}$$

$$= \frac{k_{12}k_{13}B_t^{a/4}}{K_1B_t^{b_2/4}(K_2C_{t,t})^{c_3} + K_3B_t^{b_3/4}(K_4C_{i,t})^{c_4} + \dfrac{K_5D^{b_4/4}(K_6C_{d,t})^{c_5}}{K_7B_tC^{e}_{t,d} + K_8B_i\left(\dfrac{\dot{r}_{i,t}C_{i,d}}{\dot{r}_{i,t}}\right)^f}} \quad [35]$$

and for the single-field situation (no inducing field)

$$\frac{k_{12}k_{13}B_m{}^{a/4}}{K_1B_m{}^{b1/4}(K_2C_{m,m})^{c_1} + \dfrac{K_5D^{b_4/4}(K_6C_{d,m})^{c_2}}{K_7B_mC^e{}_{m,d}}}$$

$$= \frac{k_{12}k_{13}B_t{}^{a/4}}{K_1B_t{}^{b2/4}(K_2C_{t,t})^{c_3} + \dfrac{K_5D^{b_4/4}(K_6C_{d,t})^{c_5}}{K_7B_tC^e{}_{t,d}}} \quad [36]$$

It should be noted that in Formulae 35 and 36 the introduction of exponents a, b and c, has been found necessary in fitting data; the O functions also have been specified. These terms and functions will be discussed in detail later.

The solutions of Formulae 35 and 36 were performed on an IBM 610 digital computor. The programs for these formulae are available from the author on request.

Backward inhibition. Although backward inhibition does exist, as shown by Hartline and Ratliff (1957; see pp. 364–365) it is relatively small. It is seen in their Fig. 3 that for a log inducing frequency .3 greater than the log test frequency the log reduction in the inducing frequency, through backward inhibition, was about one-fifth the log reduction in test frequency, through primary inhibition. Since test-inducing frequency disparities greater than .3 log units only produced less backward inhibition, it therefore does not seem advisable at this time to complicate our already cumbersome formulation with the many possible backward inhibition terms.

PSYCHOPHYSICAL DATA AND THEORETICAL FITS

General Design

The results to be described come from experiments the general designs of which are quite similar. To one

eye of S is presented a test field (t). For multiple-field experiments only, nearby or surrounding the test field, an additional field, the inducing field (i) is presented to the same eye. Also to this eye near the test field is usually a small fixation point which the eye fixates. To the other eye is presented a match field (m) and a small fixation point nearby. The S's first task is to fuse the left and right eye fixation points so that in the binocular view they appear as one.

The S is now asked to equate the test and match fields in brightness by adjusting either the match luminance or the test luminance. The adjusted match- or test-luminance value is the usual dependent variable which is taken as a measure of test brightness. In this general manner, test brightness has been measured for the multiple-field situation as a function of inducing area, test area, inducing luminance, test luminance, and inducing-test separation, and, for the single-field situation, as a function of test area and test luminance.

Experimental Results and Theoretical Fits

In Table 1 is presented a summary of the nine experiments to be considered and the values of the terms and functions in Equations 35 and 36 determined by each experimental situation. The procedures for determining values for the terms were as

TABLE 1

EXPERIMENTAL AND FITTED VALUES OF TERMS IN EQUATIONS 36 AND 37

Main Variable	Inducing Area		Test Area		Inducing and Test Luminance		Inducing Luminance and Separation			Single Test Area
Experimenters	Diamond I 1955	Diamond II 1955	Diamond I 1959	Diamond II 1959	Diamond 1953	Heln'n 1955	Beitel 1936	Leib'z 1954	Fry 1953	Diamond 1959
Values determined by experimental situation										
Log luminance										
B_m	v[a]	v	v	v	v	v	−0.55	.00	0.73	v
B_t	0.60	0.60	v	v	v	v	v	v	v	v
B_s	v	v	v	v	v	v	v	v	v	|
Log area-separation										
$C_{m,m}$	2.62	2.62	2.62	v	v	2.35	2.10	2.57	2.57	2.37
$C_{d,m}$	4.19	4.19	4.19	4.19	4.19	4.19	4.19	4.19	4.19	4.19
$C_{m,d}$	0.96	0.96	1.28	v	1.28	1.01	0.24	1.19	1.88	1.00
$C_{d,t}$	2.62	2.62	v	v	2.62	2.35	2.10	2.57	2.57	v
$C_{d,t}$	v	v	2.14	2.14	2.44	2.79	v	v	v	—
$C_{d,t}$	4.19	4.19	4.19	4.19	4.19	4.19	4.19	4.19	4.19	4.19
$C_{t,d}$	0.96	0.96	1.28	1.28	1.28	1.01	0.24	1.19	1.88	v
$A_{t,t}/A_{t,t}$	v	−0.48	v	v	−0.30	−0.07	v	v	2.18	|
Calculated exponents (Eq. 37)										
b_1	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00
b_2	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00
b_3	4.00	4.00	4.00	4.00	4.00	9.20	8.00	6.40	6.40	—
b_4	10.92	10.92	10.92	10.92	10.92	10.92	10.92	10.92	10.92	10.92

[a] v = variable.

TABLE 1—(Continued)

Main Variable	Inducing Area		Test Area		Inducing and Test Luminance		Inducing Luminance and Separation			Single Test Area
Experimenters	Diamond I 1955	Diamond II 1955	Diamond 1959	Diamond II 1959	Diamond 1953	Hein'n 1955	Beitel 1936	Leib'z 1954	Fry 1953	Diamond 1959
Calculated exponents (Eq. 38) c_1	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30
c_2	v	v	v	v	v	v	1.99	0.91	0.22	v
c_3	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30
c_4	v	v	v	v	v	v	v	v	v	v
c_5	0.30	0.30	v	v	v	v	v	v	v	v
Values determined by empirical fits Spont. dis. D	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Const. expon. a	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00
e	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
f	3.00	3.00	3.00	3.00	3.00	3.00	8.00	3.00	3.00	—
Log proportionality const. K_1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
K_2	−2.80	−2.80	−2.80	−2.80	−2.80	−2.80	−2.80	−2.80	−2.80	−2.80
K_3	−0.59	−0.59	−0.50	−0.50	−1.40	−22.00	0.90	−0.15	−7.85	—
K_4	−2.35	−2.35	−2.28	−2.28	−1.40	35.50	−2.00	−2.88	−1.60	—
K_5	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30
K_6	−3.75	−3.75	−3.75	−3.75	−3.75	−3.75	−3.75	−3.75	−3.75	−3.75
K_7	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
K_8	−3.40	−3.40	−3.40	−3.40	−3.40	−3.40	2.80	−3.40	−3.40	—

follows: The values of the luminance (B) terms were taken directly from the luminance values of the different fields. The area-separation (C) terms were calculated according to Formula 10 above. The dimensions of the discharge area were arbitrarily chosen; since only minimal inhibitory effects occur between fields separated by more than $4.5°$ (Fry & Alpern, 1953) of visual angle and at certainly no more than $9°$ (Leibowitz, Mote, & Thurlow, 1953), spontaneous discharge elements further than $9°$ from the center of the test field were not considered to be effective.

The value of D (spontaneous discharge activity) is arbitrarily taken as the value one. This turns out to be empirically satisfactory and must do until such time as physiological, and/or psychophysical, manipulation of discharge activity reveals its actual value.

The value of the a and K terms were determined empirically (by trial and error fitting). The values of k_{12} and k_{13} are not included in Table 1 since these terms cancel out in Equations 35 and 36. The b and c terms turn out to be variables which have been necessary for successful empirical fits, and which are specifiable in surround, luminance, and area terms. That is, b turns out to be a function of the amount by which the inducing field surrounds the test field and the ratio of the number of inducing elements to test elements, specifically

$$b = 1.72\,(5 - 4e^{(.003\,S_i\,E_i/E_t)}) + .57 \quad [37]$$

where S_i equals the proportion of the total circumference of the test field that faces or borders upon the inducing field, E_i and E_t equal the total number of inducing and test elements respectively, and e equals the Naperian logarithmic base. The values of the terms E_i, E_t, and S_i depend

upon which elements, or fields, interact. That is, if the test fibers are interacting within the test field then E_i equals E_t, S_i equals one and therefore according to Equation 37, b_2 (see Equation 35) equals four. The term b_1 involving the match field, would also equal four. If the interaction is between the test and inducing fields, however, b_3 might assume any one of a number of values because E_i, E_t, and S_i would be independent; since Equation 37 is an ogive growth function, b_3 has a lower limit of 4 and upper limit of 10.92 (see Table 1).

The c term includes the b term, in Equation 37, as well as the logarithm of the ratio of inducing to test luminance plus one, or

$$c = \left(\frac{b}{4}\right)^{1.1} \log\left(\frac{B_i}{B_t} + 1\right) \quad [38]$$

Within the test field, for example, this ratio always has to equal .3010, that is

$$c_3 = \left(\frac{b_2}{4}\right)^{1.1} \log\,(1+1) = .3010 \quad [39]$$

The term c_1 also equals .3010, whereas c_2, c_4 and c_5 all can assume independent values.

It should be emphasized that Equations 37 and 38 have been derived empirically. What physiological mechanisms they are based upon are yet to be established.

The O functions have also been empirically determined. These are as follows:

$$O_{m,d} = K_7 B_m C°_{m,d} \quad [40]$$

$$O_{t,d} = K_7 B_t C°_{t,d} \quad [41]$$

$$O_{i,d} = K_5 B_i \left(\frac{\bar{r}_{t,i} C_{i,d}}{\bar{r}_{i,t}}\right)^{f} \quad [42]$$

where $\bar{r}_{t,t}$ is the average separation between elements in the test field; $O_{i,d}$ it turns out, is a function of the ratio of within-test separation to test-inducing separation. The terms e

and f in these equations are exponents the values of which remain constant (see Table 1).

It is suspected that these O effects are based upon off stimulation which results from the combined effect of scattered light, direct light, and on-off antagonism. The functions above, however, are not derived from such mechanisms, but rather represent convenient functions which shall most likely change as we discover more about light scatter in the intact human eye, and on-off antagonism.

Experimental results. In considering each of the eight experiments to be presented, we shall mention special features of design, the dependent and independent variables, a graphic and theoretically fitted description of the results, and finally a discussion of the theoretical fit. Multiple-field data

Experiment II

Experiment I

FIG. 5. Stimulus patterns for study of inducing-area variation. (After Diamond, 1955.)

will be considered before single-field data.

Multiple-field data: inducing area. Diamond's article (1955) is the only one to be considered with respect to inducing-area variation. Actually, two experiments were performed. Both employed the above general design and both employed match luminance as the dependent variable and inducing area and luminance as independent variables. The sizes of the separate fields (in visual angle) are indicated in Fig. 5. Experiment I and II differed in the manner in which the inducing area was manipulated. In I, inducing and test borders were held adjacent. In II, the distance between the centers of the inducing and test fields was held constant. The purpose was to test whether this change in inducing-area variation would effect a change in the data from Experiment I to II in the precise manner that would be dictated by our theory, i.e., when the necessary changes were made in $r_{i,t}$ and therefore in $C_{i,t}$ in Formula 35. The success of a theoretical prediction of this change is seen in a comparison of the theoretical fits of two sets of data taken from one S, AD, see Fig. 6. The only values that change in Equation 35 from one fit to the other are those dictated by the change in $C_{i,t}$; otherwise all constants and other terms remain the same as one would expect with the same individual and apparatus, see Table 1.

Probably more important, of course, is the fact that Formula 35, based upon elemental physiological inhibitory effects, can predict the course of psychophysical depression as inducing area changes.

It should be noted that the spontaneous discharge term (see Formulae 20 and 35) does not change enough to show any appreciable enhancement
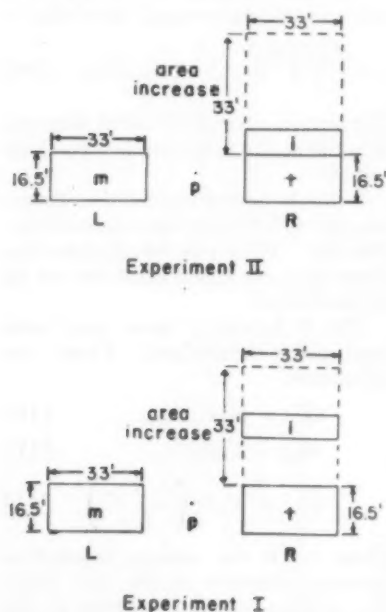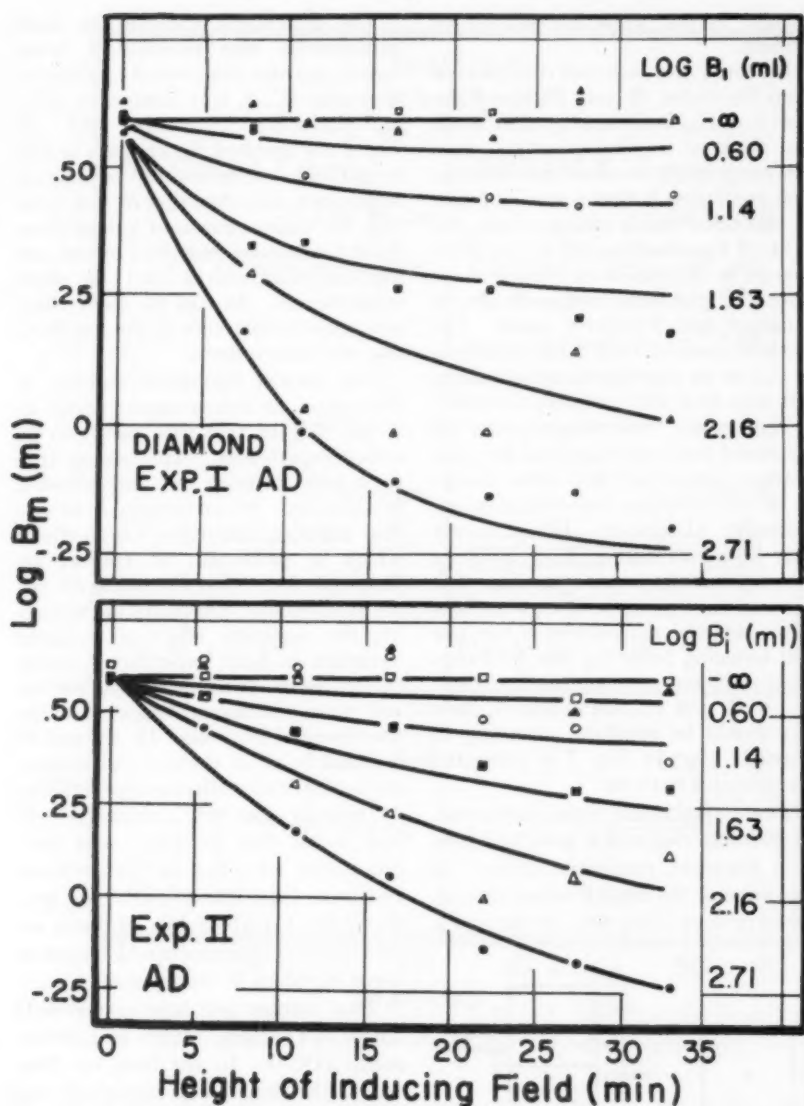
FIG. 6. Test brightness as a function of inducing area. (Diamond, 1955.)

effect. That is, $O_{i,d}$, of which this term is a function, is not very large at maximum (since $C_{i,d}$ is effectively small as we shall see later) and does not change very much with respect to $O_{t,d}$; enhancement, therefore, which according to our theory comes from O or off inhibition of spontaneous dis-

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charge, is not great enough to be evident.

It should also be noted that, as seen from Formulae 38 and 39, the exponent c_1 of $C_{i,t}$ is always equal to .3010. An obvious prediction arises from this empirically required relationship. The prediction is that a manipulation of test area which changes only the value of $C_{i,t}$ should result in very little change in depression or enhancement since the exponent of $C_{i,t}$ is always constant and relatively small. Let us then consider next a manipulation of $C_{i,t}$ in an experiment which varies test area in a multiple-field situation.

Test area. This experiment by Diamond (not yet submitted for publication) employed the same design as in his foregoing inducing-area experiment (Diamond, 1955) except that test area rather than inducing area was manipulated. In order that $C_{i,t}$ not vary along with $C_{i,t}$, the distance between the centers of the test and inducing fields $r_{i,t}$ was held constant while test area was manipulated; i.e., if E_i and $r_{i,t}$ are constant, then $C_{i,t}$ should be constant according to Formula 10. In Fig. 7 is presented the stimulus pattern.

Two experiments were performed. In the first the match area was held at a constant maximum value. In the second the match area was allowed to vary along with the test area.
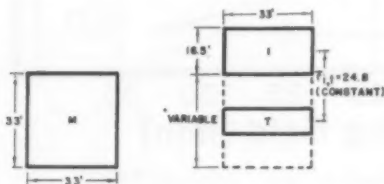
FIG. 7. Stimulus pattern for test-area study. In Experiment I the match field is at a maximum height of 33'. In Experiment II the match-field height always equals that of the test field. (Diamond, unpublished.)

The dependent variable for both experiments was match-field luminance and the independent variables, test area ($C_{i,t}$), test luminance (B_t), and inducing luminance (B_i). In Fig. 8 are graphed the changes in test brightness, as measured by match luminance, as a function of test area, $C_{i,t}$, for various constant values (constant for any one function) of test and inducing luminances and for both experiments. As can be seen, there is not much difference in the results of the two experiments.

Nor does a significant amount of depression or enhancement occur as a function of test-area variation in either experiment. This seems true for a wide range of test and inducing luminances. It can be seen, however, that inducing luminance has an effect, which is predicted, of course, by Equation 35. That Formulae 38 and 39, particularly, are useful in predicting the negligible effect of test-area variation in both experiments seems to be borne out. Actually, further and more widespread support for the usefulness of Formulae 35, 38, and 39 is found in all of the successive theoretical fits of the different experiments.

Inducing and test luminances. It was found that inducing- and test-luminance variation in the previous test-area experiment could be predicted by Equation 35. Suppose we now turn to experiments which explore these variables in more detail.

Two studies are here considered: Those of Diamond (1953) and Heinemann (1955). In the first, by Diamond, the design was essentially the same as in his area experiments, except that now, inducing and test luminances rather than areas are varied independently. Match luminance is again the dependent response. Fig. 9 describes the stimulus pattern and
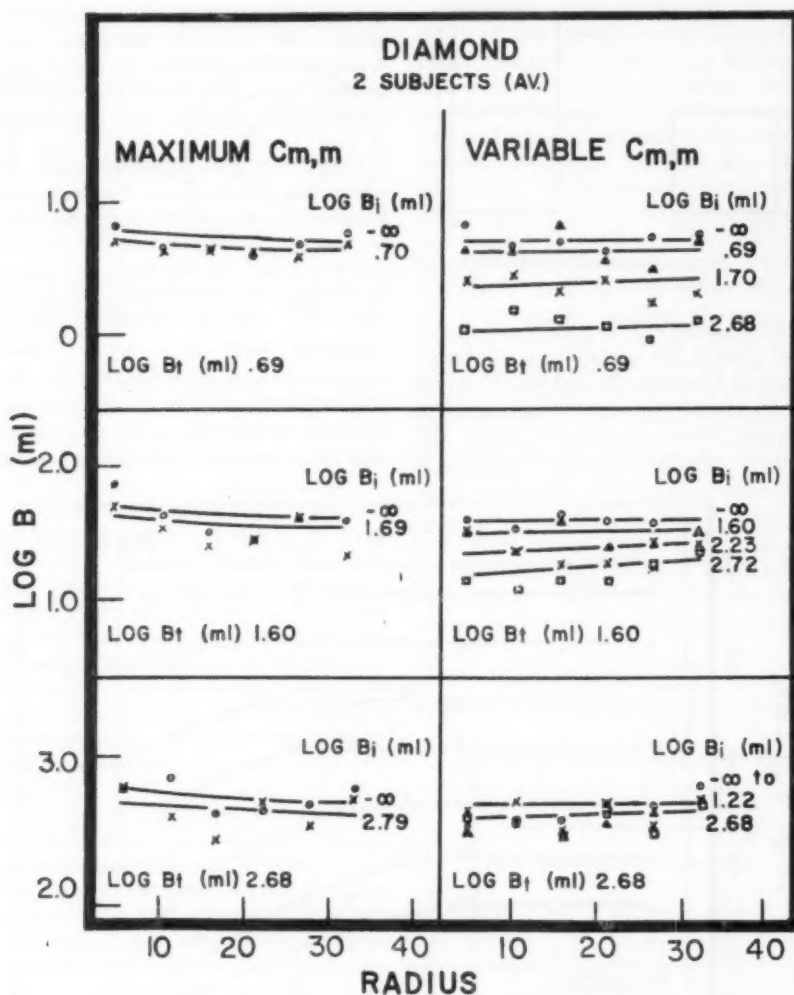
FIG. 8.   Test brightness as a function of test area.   (Diamond, unpublished.)

Fig. 10 the results, theoretically fitted again with Equation 35.

It can be seen in Fig. 10 that the effect of inducing inhibition only becomes apparent after inducing luminance exceeds test luminance, or when inducing inhibition becomes large with respect to within-test inhibition.

Also, only depression is significantly evident; there seems to be no enhancement (which should occur as a rise in test brightness above isolated test brightness). Again, the lack of
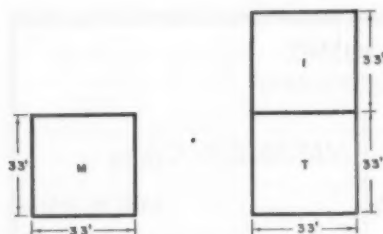
FIG. 9. Stimulus pattern for study of inducing and test luminances. (Diamond, 1953.)

enhancement can be attributed to the fact that in the spontaneous discharge term in Formulae 20 and 35, the value of $O_{i,d}$ never changes enough, relative to the values of $O_{t,d}$ to make the entire spontaneous discharge term change enough to produce enhancement. Let us now, therefore, pay experimental attention to this term to see if we can make $O_{i,d}$ change significantly, relative to the value of $O_{t,d}$.

There are two obvious ways in which this can be done. First we can increase the area of the inducing field so that it is many times the size of the test field. In this way, according to Formula 42, the effects of B_i in the term $O_{i,d}$ will be amplified by a relatively large $C_{i,d}$ term. Thus, changes in $O_{i,d}$, as a function of B_i, will be larger relative to $O_{t,d}$ values. The second way of making $O_{i,d}$ variation large relative to $O_{t,d}$ is to make
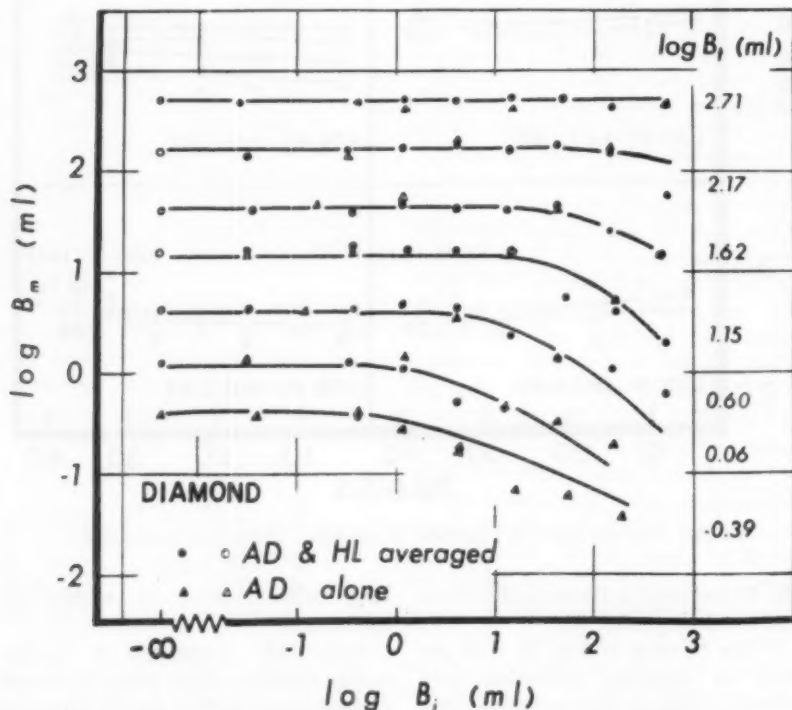
FIG. 10. Test brightness as a function of test and inducing luminances. (Diamond, 1953.)
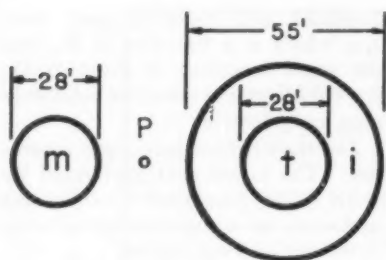
FIG. 11. Stimulus patterns for study of test and inducing luminances. (Heinemann, 1955.)

B_t small (see Formula 41). Thus, an increase in $O_{i,d}$, relatively large with respect to $O_{t,d}$, should decrease the effectiveness of spontaneous discharge and produce enhancement.

Let us consider the first possibility of increasing $O_{i,d}$ through an increase in $C_{i,d}$. An experiment in which $C_{i,d}$ is very large is that of Heinemann (1955) who like Diamond (1953) explored test brightness (measured by match luminance) as a function of inducing and test luminances. Unlike Diamond, however, Heinemann's inducing area was much larger than, and surrounded, his test area (see Fig. 11).

In Fig. 12 are presented the theoretically fitted data of Heinemann. As in Diamond's results (1953), it is seen that depression begins to occur approximately at the point at which inducing luminance equals and exceeds test luminance. The depressive effect, as inducing luminance increases, is much greater in Heinemann than in Diamond, as one might expect, since Heinemann's inducing field is much larger; a larger $C_{i,t}$ in Formula 35 multiplies the effectiveness of the B_i change.

With respect to our prediction, that a large $C_{i,d}$ value will bring out enhancement as B_i is increased, we find that this does occur in the data as well

as in our theoretical curves. If we think of this at the retinal level, we might describe the results as follows: When there is no inducing field present (B_i equals zero) the isolated test field is being inhibited by the spontaneous discharge activity in the surround. As the inducing luminance increases, if $C_{i,d}$ is large enough, it now begins, through off inhibition, to suppress spontaneous discharge, thus disinhibiting the test field and allowing it to appear brighter than when isolated. At this point, of course, the inhibition of the test field by the on fibers in the inducing field, is not yet great enough to produce a net depressive effect. However, as inducing luminance increases further, spontaneous discharge becomes completely depressed, and the inducing on inhibition now "takes over" the function so that only depression is apparent; this can be seen for the high inducing (relative to test) luminance values in Fig. 12.

We should also note that as test luminance and match luminance in-
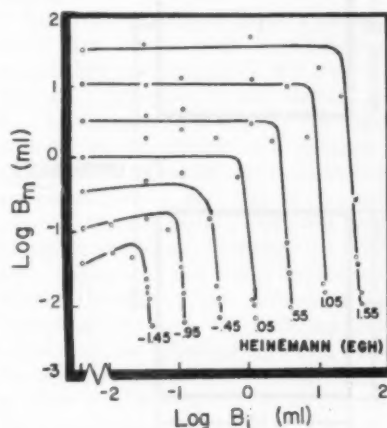

FIG. 12. Test brightness as a function of test and inducing luminances. (Heinemann, 1955.)

crease together, enhancement decreases, since the spontaneous discharge term becomes less effective. This is seen in Fig. 12; only the three bottom theoretical curves show enhancement. This decrease in enhancement with increased test luminance is also seen in Heinemann's data, except that the four bottom data curves show enhancement, which enhancement then disappears as predicted for the higher curves. Otherwise the theoretical fit to the data seems satisfactory.

Let us now consider the second way of making $O_{i,d}$ variation large relative to $O_{t,d}$, and therefore of allowing changes in spontaneous activity to bring out enhancement. This second method involves making $O_{t,d}$ relatively small; this occurs in an experiment by Beitel (1936) who measured inducing effects at threshold test luminances. That is, since B_t was at

Fig. 13. Stimulus pattern for study of test-threshold luminance and test-inducing separation. (Beitel, 1936.)

threshold and therefore very low, $O_{t,d}$, which is a function of B_t, was also small according to Formula 41; $O_{i,d}$ variation was therefore relatively large and effective.

Threshold-test luminance and separation. The experiment performed by Beitel (1936) measured threshold-test luminance as a function of inducing luminance and separation.

Beitel's design was generally similar to that of Diamond (1953). His test and inducing fields were squares, 10' on a side (see Fig. 13). He used no match field since his measure was of test threshold. The separation between the centers of the test and inducing fields varied from 3.5' to 37.5'. Thus when the Ss were centrally fixated, stimulation was foveal.

Beitel achieved fixation by presenting a dimly illuminated 8° field at the center of which the Ss were instructed to look. At the center of this field the test and inducing fields were then simultaneously exposed for .02 sec. for each test threshold judgment. The use of this 8° fixation field seems unfortunate. Not only does it make a theoretical analysis more complicated, but, since the luminance of the 8° field was not specified, an exact theoretical account becomes impossible. Actually, this fixation field could act as an additional inducing field, but to what extent we cannot say.

Fig. 14 presents the theoretically fitted results. It is found that although the general shape of the curve has been fitted, the values of the constants b_3, K_8 and f in Table 1 are relatively high; b_3 should be equal to four according to Formula 37, log K_8 equal to -3.4 and f equal to three according to other experiments. Since, however, the low luminance 8° fixation field could act as an additional inducing field, the increases in these constants are actually to be expected.
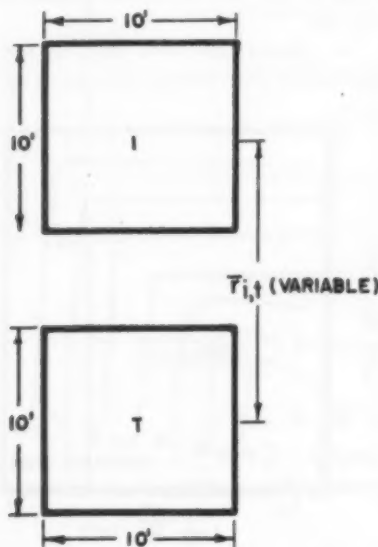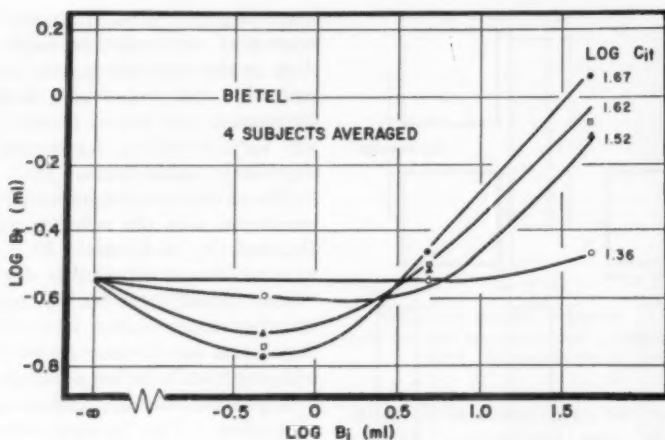
FIG. 14.   Test-threshold luminance as a function of inducing
luminance and test-inducing separation.   (Beitel, 1936.)

Despite these increases, however, the general shapes of the theoretical and empirical curves are the same as that of Heinemann's (1955), as our theory predicts. It should be noted that Beitel's curves are inverted with respect to those of Heinemann; that is, Beitel's experiment involved a constant test-threshold brightness and measured test luminance as the dependent variable; Heinemann's response involved a constant test luminance and measured test brightness (as indicated by match luminance) as the dependent variable.

Thus, in Beitel as in Heinemann, enhancement results for low inducing luminances, and depression results for high inducing luminances. Or in the terms of our theory, if spontaneous discharge is appreciably inhibited by slight inducing-field luminance, enhancement will occur. Such an inhibition requires that the values of $O_{i,d}$ in Equation 20 be large relative to those of $O_{t,d}$. In Heinemann $O_{i,d}$ is relatively large because $C_{i,d}$ is relatively large (see Formula 42). In Beitel $O_{i,d}$ is relatively large because

B_i and therefore $O_{i,d}$ is relatively small. In both experiments, therefore, enhancement should theoretically occur, and as seen in Figs. 12 and 14 actually does. In both experiments, moreover, depression occurs when inducing luminance becomes very large relative to all other terms.

One final aspect of Beitel's experiment has to do with test-inducing separation as measured by $r_{i,t}$ in Fig. 13. According to our theory, as $r_{i,t}$ increases there should be two general consequences: (a) enhancement should decrease, since according to Formulae 35 and 42, as $r_{i,t}$ increases, $O_{i,d}$ variation becomes less effective and thus discharge activity less changed; and (b) depression should decrease, since according to Equations 10 and 35, inducing-test inhibition, as affected by $C_{i,t}$, decreases with an increase in $r_{i,t}$. Both of these predictions are borne out in Beitel's data in Fig. 14.

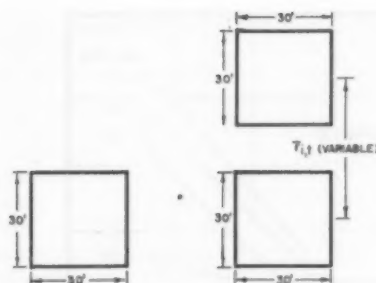The predicted effect of separation occurs, therefore, at threshold luminances of the test field. Suppose,

FIG. 15. Stimulus pattern for study involving inducing luminance and test-inducing separation. (Leibowitz et al., 1953.)

involving $\mathcal{T}_{i,t}$. But, if we use equal-sized test and inducing fields (as is done in the experiment next to be described) and relatively high test luminances, we would expect that as we vary inducing luminance, only depression would occur.

The reason for this, as stated above, has to do with the relative values of $O_{i,d}$ and $O_{t,d}$ in Formula 20. At any rate, we should expect that depression at least should occur for high inducing-test luminance ratios and that this depression should decrease with an increase in test-inducing separation, $\mathcal{T}_{i,t}$.

Suprathreshold test luminance and separation. Two investigations will be described, the first by Leibowitz, Mote, and Thurlow (1953) and the second by Fry and Alpern (1953). In the first by Leibowitz et al., the stimulus pattern was the same as that of Diamond (1953) except that the fields of Leibowitz et al. were slightly
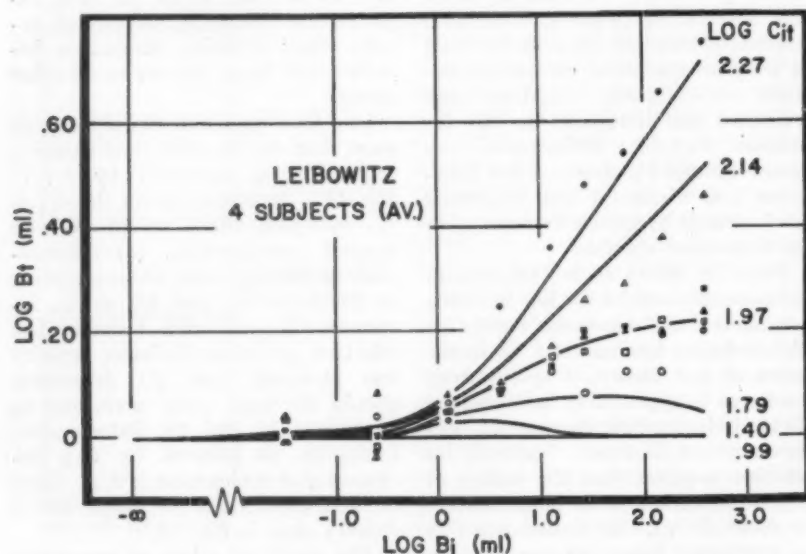
however, we experiment with supra-threshold test luminances, with a technique similar to that of Beitel. Will a variation in test-inducing separation $\mathcal{T}_{i,t}$ still be effective? The theory would predict, of course, that should enhancement or depression occur, these effects would be reduced according to the same formulae above

FIG. 16. Test luminance as a function of inducing luminance and test-inducing separation. (Leibowitz et al., 1953.)

smaller; i.e., 30' on a side (see Fig. 15). Unlike Diamond, however, Leibowitz et al. employed test luminance as a measure of his dependent response. His Ss adjusted test luminance until test brightness equalled that of the match field, which was kept at a constant luminance and thus constant brightness level. The test luminance necessary to maintain a constant brightness effect was therefore being measured. Note that this is similar to a threshold measure, as in Beitel (1936), which records the test luminance necessary to maintain the threshold effect, which effect is presumed constant.

The results of Leibowitz et al. are graphed in Fig. 16. As predicted, no enhancement is clearly evident. Further, the depression that occurs for high inducing luminances becomes weaker as test-inducing separation increases. Although these major changes are predicted by our theoretical curves it should be pointed out that the theoretical fit is far from an exact one. Further, the value of b_3 (see Table 1) is 2.4 higher than that predicted by Equation 37. Why this is so is not immediately obvious. One might speculate, however, that the fit is thrown off by the fact that the inducing field moves out into the periphery for separations greater than 60'. Alpern (1953) has shown that peripheral stimulation results in greater depression than foveal stimulation. The attempt, therefore, to apply the same theoretical function to all of Leibowitz's data might have resulted in the generally poor fit that we find in Fig. 16.

A final experiment which explored test-inducing separation at suprathreshold test luminance is that of Fry and Alpern (1953). Their fields were larger than those of Leibowitz et al. (2.5° × .5°), such that when S
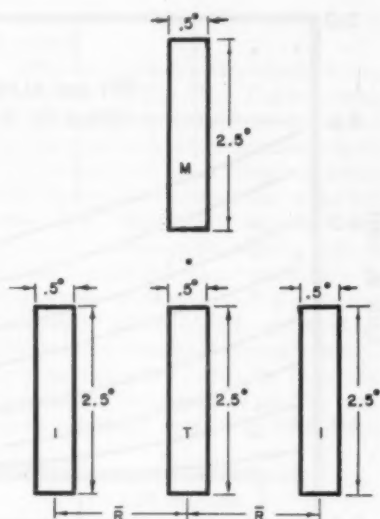

FIG. 17. Stimulus pattern for study of inducing luminances and test-inducing separation. (Fry & Alpern, 1953.)

fixated the fixation point, part of the test field extended into the parafovea and resulted in possible peripheral as well as foveal test stimulation (see Fig. 17). In addition, they employed two inducing fields, rather than one and these definitely stimulated the periphery. As in Leibowitz, the dependent response was test luminance (match luminance constant) which was measured as a function of test-inducing separation and inducing luminance.

Their results are graphed and theoretically fit in Fig. 18. As can be seen, no enhancement is theoretically predicted nor occurs. This is mainly because inducing luminance is always equal to or greater than test luminance; thus inducing luminance starts at a high enough value to produce a net inhibitive effect, overbalancing whatever enhancement is occurring.

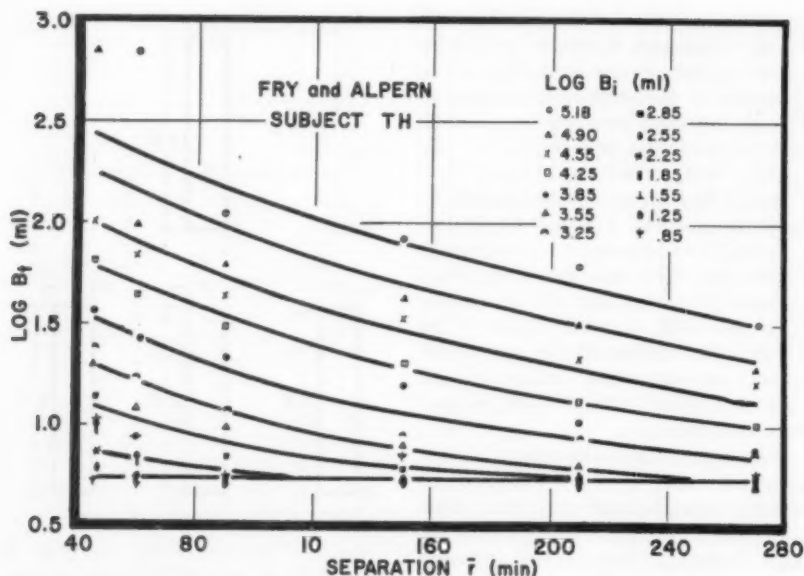As in Leibowitz's experiment, the value of b_3 for this experiment should

Fig. 18. Test luminance as a function of inducing luminance and test-inducing separation. (Fry & Alpern, 1953.)

be noted in Table 1. It is 1.7 higher than that value predicted by Equation 37. It is probable that this increase in b_3 in both experiments is caused by the fact (determined by Alpern, 1953) that peripheral stimulation produces a greater depression than foveal stimulation; since our equations were built around foveal data, the discrepancy we get in b_3 is in the predicted direction. Eventually, of course, we will want to include a term in our equations to represent the location of retinal stimulation.

Single-field data: Area and luminance. As has been pointed out above, the results of area and luminance variation in a single test field are of particular interest to our theory and should be predicted by it. A number of experiments are applicable here, although only one will be considered because the others lacked certain controls. That is, Hanes (1951) investigated test brightness as a function of test area at suprathreshold luminance levels. Unfortunately, he did not attempt to control adaptation nor fixation in his procedure. Graham, Brown, and Mote (1939) studied test-threshold luminance as a function of test area. They employed a dimly illuminated annulus (surrounding the test area) for fixation; the presence of this annulus, however, might have affected test thresholds, since the annulus luminance was at times greater than test-threshold luminance.

In Diamond's study (not yet submitted for publication) which covered both suprathreshold and threshold-test luminances as a function of test area, these objections are met. Adaptation is procedurally controlled and fixation is achieved with the use of a 3' fixation point, relatively small
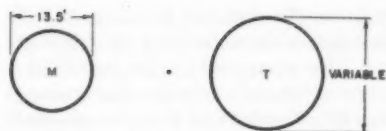
FIG. 19. Stimulus pattern for study of single-field test luminance and area. (Diamond, unpublished.)

separated by 17.2' (from border to border).

Diamond's design employed circles, as pictured in Fig. 19. The test field, which varied in area, was presented to the right eye, and the match field, which was held at a constant intermediate area, was presented to the left eye. The dependent variable was the test luminance required either to match a constant match luminance in brightness or to produce a constant threshold effect. With respect to the threshold results, a special assumption is necessary; this is that the match-field luminance is at this constant threshold level. In fitting the

and/or at a noninhibiting distance from the test area. That is, when the test area was almost as small as the fixation point, it was at a distance of 21' from it. The value of 21' seemed reasonable since Beitel (1936) found no interaction in equal areas
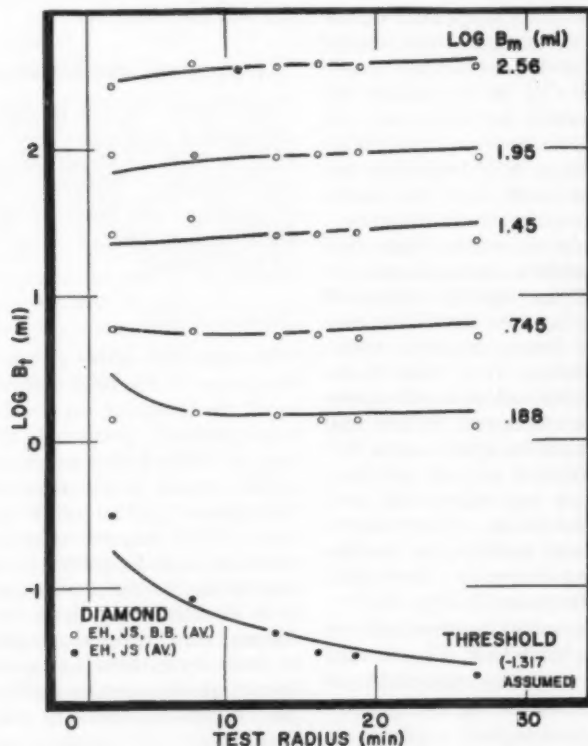

FIG. 20. Test luminance as a function of test area and match luminance. (Diamond, unpublished.)

threshold curve, therefore, the match "diameter" was given the same intermediate value (13.5′) as for the suprathreshold curves, and "luminance" was assumed equal to the test-threshold luminance for that particular diameter. The result of this, of course, is that the left side of Equation 36 remains constant, which condition reflects the fact that the match field is at a constant "effect" level (of threshold).

As can be seen in Fig. 20, the theory predicts the data in a satisfactory manner. At the low test-threshold luminance values, enhancement takes place. This is to be expected from Equation 36, since at low B_t values the spontaneous discharge term which has B_t in its denominator (see Equations 20, 36 and 41), is relatively large; variation of $C_{t,d}$ in Equation 36, therefore, should be effective. At high B_t values, however, the spontaneous discharge term is already too small for variation over the same $C_{t,d}$ range to make much difference. Or, in speculative words, high test luminance inhibits spontaneous activity by direct and/or scattered light so that as test area increases there is little or no change in spontaneous activity. Low test luminances, however, allow spontaneous activity to be effective. At low test luminance therefore, spontaneous discharge inhibition is present and thus reduced by an expanding test area through off inhibition. Test brightness, in this way increases, or threshold luminance decreases. Such predictions are borne out in Fig. 20.

It should be noted that variation of internal test inhibition, $C_{t,t}$ is not very effective since its exponent c_3 is equal to .3010, according to Equation 39. The physiological explanation for this lack of effect is not obvious at this time. We might point, however, to the data of Boynton (1951) to support the reliability of this finding. He measured the height of the b wave in electroretinographic recordings as a function of the area of small test fields. He found that as area increased, the b wave response also increased. The interesting point, however, was that he could account for the charge in b wave height solely on the basis of the change in the amount of scattered light hitting the retinal surround of the test field. Presumably, changes of the area of light striking within the test borders neither reduced nor increased the b wave response. Which is to say in our own theoretical terms, that variation of $C_{t,t}$ was not effective.

DISCUSSION AND CONCLUSIONS

Our theory set out to explain psychophysical depression and enhancement in terms of physiological inhibition occurring between on fibers (depression) and between on and off fibers (enhancement). How successful has it been?

Negative Aspects

We find that although the changes that occur (in the brightness response) in all the foregoing experiments have been generally predicted, it can be seen in Table 1 that exact prediction of the results in the experiments of Heinemann (1955) and Fry and Alpern (1953) require values of constants K_3 and K_4 quite a bit different than those for the other experiments. It is doubtful that these changes in K_3 and K_4 can be attributed to uncontrolled variables (e.g., personality) that might be operating differently in the Heinemann and Fry and Alpern experiments. It seems more likely that our theory is lacking in its development of the term S_i in Formula

37. That is, that amount by which the inducing field surrounds the test field is substantially greater in the Heinemann and Fry and Alpern experiments than in the others. Before this term is developed in detail, however, it would probably be more appropriate to first know empirically what happens to test brightness as S_i is varied.

Another consideration is that our b and c equations (37 and 38) are not rational. Nor are the O formulae (40, 41, 42) rational, even though we can point to their probable physiological bases. The improvement of this situation also awaits further experimentation and knowledge.

Positive Aspects

On the positive side of the picture, our theory has integrated in an explicit fashion all the major changes that occurred in the physiological experiments and psychophysical multiple- and single-field experiments that were considered. And except for changes in K_3 and K_4 the empirically determined constants remain relatively stable, except when, because of a particular experimental procedure or lack of control (as in Beitel, 1936) a change in constants is predicted.

Predictions

An interesting prediction is suggested by the foregoing theory. This has to do with the relationship between enhancement and spontaneous discharge and the possible dependence of spontaneous discharge upon reticular activation. According to our theory, changes in reticular activation could change the value of the term D in Equation 35 and, therefore, change the amount of enhancement present in either multiple- or single-field situations. Preliminary experiments indicate that this might be happening.

An initial experiment by Diamond is investigating the correlation between the amount of single-field enhancement (as found in Diamond's single-field area experiment above) and the amount of reticular activation, as measured by blood pressure— Lindsley (1951) suggests this as a possible measure. Our theory would predict that the higher the reticular activation as measured by blood pressure, the greater the amount of single-field enhancement. The results, taken from a sample of 30 college students, so far show a positive and significant (at the .01 level of confidence) Spearman rank order correlation between enhancement and blood pressure as predicted. The correlation is low but it still suggests support of our theory; i.e., factors other than reticular activity might be causing blood pressure differences from S to S, thus reducing the correlation. This experiment has yet to be performed for the multiple-field situation.

A second investigation by Kleman, Diamond, and Smith is studying the relationship between enhancement and the administration of a drug that might affect reticular activation. The drug that is being used is caffeine; it is a stimulant and also safe to use with college Ss. Heinemann's (1955) multiple-field situation is employed (see Fig. 11) in which enhancement appears for the low inducing- and test-field luminances (see Fig. 12). Our initial findings, in Fig. 21, show that caffeine administration causes enhancement to disappear.

Alternative theoretical explanations seem possible at this point. A first hypothesis is that caffeine stimulates the excitatory portion of the reticular system; this stimulation is relayed to the retina, where, as a result, on fiber activity is increased more than is off fiber activity. Granit (1955, pp. 106–

107) has shown that on as well as off fiber activity increases with reticular stimulation. A relatively strong increase in on activity would weaken off activity and reduce enhancement.

A prediction accompanying this hypothesis is that caffeine administration should lower absolute threshold for a single field (since on fiber ac-

tivity is supposed to increase). We find in a preliminary experiment that this does happen for some Ss but not for others. Why these individual differences exist in this respect is now being explored. An alternative hypothesis is that caffeine acts directly on the on fibers in the retina to reduce enhancement in the above manner. A choice of the best of these two hypotheses awaits further experimentation.
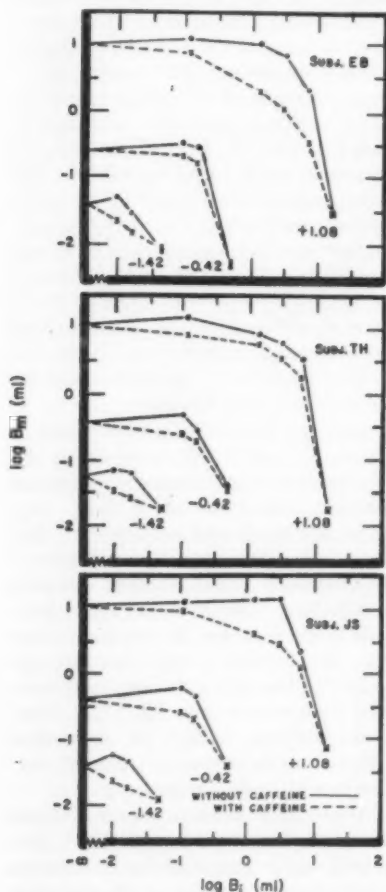

FIG. 21. The effect of caffeine on the test-brightness versus inducing-luminance function. Note that enhancement disappears with caffeine administration. (Kleman, Diamond, & Smith, unpublished.)

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A TWO–FACTOR THEORY OF INHIBITION [1]

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Perhaps no other aspect of Hull's (1943) learning theory has been so thoroughly explored, both empirically and theoretically, during recent years as his theoretical treatment of extinction and related phenomena. No fewer than 12 theoretical papers dealing with this problem have appeared in the *Psychological Review* during the past seven years (Cotton, 1955; Ellis, 1953; Estes, 1955; Glanzer, 1953a; Gleitman, Nachmias, & Neisser, 1954; Johdai, 1956; Maatsch, 1954; Razran, 1956; Saltz, 1953; Walker, 1958; Wasserman, 1952; Wolpe, 1952).

Gleitman, et al. (1954) presented a critical and important evaluation of Hull's formulation in which they pointed out several paradoxical consequences which can be derived from Hull's postulates, one being that the performance curve should rise to a maximum and with continued reinforced trials eventually decline to its starting point; another being that reconditioning to the original level would be impossible. Most of the criticisms of these authors were concerned with *conditioned inhibition,* a concept which many psychologists have held in low esteem for years.

The most serious attempt at formulation of an alternative theory has been made by Glanzer (1953a), who as-

sumed that with continued exposure to the same stimulus the organism becomes less active, i.e., stimulus satiated. Glanzer makes the additional assumption that this "boredom-like" effect dissipates with time. From this theory are deduced spontaneous alternation and related phenomena.

A critical evaluation of Glanzer's formulation, together with numerous empirical studies, illustrates the limitation of the stimulus satiation concept. Glanzer, like Hull, can explain certain empirically known facts, but finds it difficult to handle others. For instance, it is difficult to see how Glanzer's formulation can explain such a phenomenon as an increased proportion of left turns following a forced right turn and an increase in this effect following two forced right turns (Grosslight & Ticknor, 1953; Thompson, 1952). Glanzer (1953a) also failed to verify Deduction 5 which appears to be basic to his theory. Deduction 5 states: "In the simple two-alternative situation, as stimuli differentiating the alternatives are eliminated, spontaneous alternation will decrease" (p. 262).

In their treatment of inhibitory potential both Hull and Glanzer neglect several variables which may have considerable influence upon the development and dissipation of inhibition. The diverse results of the reported studies appear to be complicated by differences in such variables as the nature of the first trial in a block of two consecutive trials (free or forced) and reward for both alternatives or no reward in the maze. Both of these authors failed to differentiate between

reinforced and nonreinforced responses on inhibitory potential. The majority of studies cited in support of the stimulus satiation theory appear to employ nonreinforced responses. If this variable should lead to results which differ from studies using reinforced responses, we might find that both Hull's reactive inhibition postulate and Glanzer's stimulus satiation postulate have merit.

In view of the conflict between empirical data and the deductions from existing theories of inhibitory potential, a need is indicated for a more comprehensive theory than the one presented by Glanzer, one which also avoids the paradoxes derived from Hull's formulation.

The two-factor theory of inhibitory potential presented in this paper was first designed in an attempt to explain some of the apparently conflicting results of studies on spontaneous alternation. The theory is a modification of Hull's reactive inhibition postulate and Glanzer's stimulus satiation postulate. This revised formulation should not only enable us to take care of some of the contradictions in the theories presented by Hull and Glanzer, but yield novel predictions, and help to explain empirical results from a number of apparently unrelated areas.

Following the procedure of Hull and Glanzer, the theory is presented in the form of three postulates, as follows:

Postulate 1. Whenever an organism makes a response,[2] there is developed an increment of inhibition which decreases the probability of recurrence of that same response. This condition

[2] The term response as used here refers to acts producing changes in the immediate environment or in the relation of the organism to the immediate environment. Examples of response as defined above would be turning right in a maze, approaching a food cup, pressing a bar, running down a maze alley, etc.

is called response inhibition (I_R). The amount of I_R developed by a series of response evocations is a negative growth function of the number of evocations, and dissipates as a simple negative growth function of time.

Postulate 2. Whenever an organism makes a response, or is exposed to a stimulus in the absence of primary reinforcement, there is developed an increment of inhibition which decreases the probability that the organism will return to or respond to the same stimulus, providing the organism has previously been rewarded in the presence of similar stimuli (e.g., in previous maze training or during preliminary training). This condition is called stimulus inhibition (I_S). The amount of I_S developed is a negative growth function of number of stimulus exposures. Stimulus inhibition dissipates very slowly with the passage of time and will generalize from one stimulus to other similar stimuli.

Postulate 3. Response inhibition and stimulus inhibition summate functionally to produce total inhibition (I_T).[3] Implicit in the postulates are the following statements:

1. Whenever a response is made and followed by reinforcement, only I_R will develop.

[3] Taking the modification of Hull's theory (as presented by Spence, Goodrich, & Ross, 1959) as a point of departure the above three inhibitory constructs could be incorporated as follows:

In a simple two-alternative situation when one alternative is rewarded, the excitatory potentials of the responses to the two discriminanda $(S_+ S_-)$ are given by the following equations:

$$E_+ = H_+ \times (D + K) - I_R$$
$$E_- = (H_- \times D) - I_T$$

where $H = fN_R$ (number of trials run), $D = fTd$ (time of food deprivation), $K = fWg$ (magnitude of reward), $I_R = fN_R$, $I_S = fn$ (number of nonreinforced trial runs), and $I_T = I_R + I_S$.

2. Whenever an organism is exposed to a stimulus in the absence of reward, only I_S will develop. This is based on the assumption that the organism is not required to make a specific response in arriving at the stimulus.

3. Whenever a response is made and not followed by reinforcement, both I_R and I_S will develop.

4. When responses are learned under conditions of partial reinforcement, both types of inhibition will develop, I_R on every trial and I_S on the nonreinforced trials.

The postulates yield a large number of deductions which are relevant to several experimental situations. Some of the more basic of these are formulated and discussed below. They are divided for convenience into two main categories: deductions relevant to spontaneous alternation, and deductions relevant to extinction and related phenomena. The former are further divided into situations of constant stimuli and situations of reversed stimuli.

DEDUCTIONS RELEVANT TO SPONTANEOUS ALTERNATION

In a simple two-alternative situation, such as a **T** or **Y** maze, where the subject (S) is started from the same starting box on two consecutive trials, thus maintaining constant stimuli, the following deductions apply:

Deduction 1: Nonrewarded Ss will alternate more than rewarded Ss.

Since only I_R is developed when a response is followed by reward, and both I_R and I_S develop when a response is not rewarded, it would follow that the total amount of inhibition against repeating the turn and approaching the stimulus would be greater in the latter case. A verification of this deduction was made by

Thompson [4] who found that nonrewarded Ss showed significantly more alternation (73%) than rewarded Ss (41%).

Deduction 2: There will be a decrease in alternation with an increase in intertrial interval. This decrease will be greater for rewarded Ss than for nonrewarded Ss.

The two-factor theory assumes that I_R dissipates much more rapidly than I_S. Therefore rewarded Ss will show greater decrease in alternation after a passage of time since all of the inhibition will dissipate, whereas the nonrewarded Ss will lose their I_R but less of the I_S, thus showing less decrease in alternation after a passage of time.

Solomon (1948) and Dember & Fowler (1958) cite many studies verifying the first part of this deduction. The second part of the deduction was verified by Thompson (see Footnote 4) who found that nonrewarded Ss showed a 16% decrease in alternation with 30-min. intertrial interval while rewarded

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[4] Eighty rats were run on a cross-shaped rotating maze with two starting boxes in an attempt to study alternation as a function of three variables: (*a*) similar (black-black) vs. dissimilar (black-white) alleys and end boxes, (*b*) reinforcement in both end boxes vs. no reward in either end box, and (*c*) the first trial in each block of two being forced or free. By using a $2 \times 2 \times 2$ factorial design it was possible to determine the relative importance of each of the three main variables as well as possible interactions.

Each S was given four trials, in blocks of two, each day for 12 days. The two trials within each block were given with a 30-sec. intertrial interval and the interblock interval was 30 min. On half of the 24 blocks the Ss were given their two trials from the same starting box, either S_1 or S_2, thus maintaining a constant position of maze arms. On the other 12 blocks, the Ss were started in the S_1 (or S_2) box for trial one and then placed in the other box for trial two, thus reversing the relative position of the maze arms.

Ss showed a 61% decrease with the same intertrial interval.

In a simple two-alternative situation, such as a cross-shaped maze, when the S is started from opposite starting boxes on two consecutive trials, thus reversing the stimuli, the following deductions apply:

Deduction 3: Rewarded Ss will alternate turns and repeat stimuli.

This deduction follows from the assumption that only I_R develops when responses are rewarded. Since no I_S is developed, there will be no tendency for the Ss to alternate stimuli. This implies that Hull's reactive inhibition postulate is essentially correct if applied only to situations in which both alternatives are rewarded. This deduction has not been verified.

Deduction 4: Nonrewarded Ss will alternate stimuli and repeat turns when the maze arms are dissimilar.

The development of I_R would lead to the prediction that the Ss would alternate turns. However, I_S will develop and accumulate and eventually overcome the effect of I_R; i.e., the total amount of I_S after a few trials will be greater than the amount of I_R, which will dissipate between blocks of trials.

Glanzer (1953b), using a black-white maze found 81% alternation of maze arms (stimuli) when his Ss were started from opposite starting boxes on two consecutive trials. Thompson (see Footnote 4) found nonrewarded Ss run on the black-white maze showed 60% alternation of stimuli.

Deduction 5: Nonrewarded Ss will alternate turns and repeat stimuli when the arms are similar.

Since I_S is assumed to generalize to similar stimuli, the I_S developed to Arm A will generalize to Arm B and since the amount of I_S is approximately equal for both of the alternatives, the Ss response will be determined largely by I_R. Thompson (see Footnote 4) found verification of this deduction in the fact that nonrewarded Ss run on the black-black maze alternated turns 65% of the time as compared to only 40% turn alternation on the black-white maze.

Deduction 6: Rewarded Ss will show a decrease in turn alternation and an increase in alternation of stimuli with an increase in intertrial interval.

The stimulus satiation theory makes the opposite prediction, i.e., a decrease in alternation of stimuli with an increase in intertrial interval since Glanzer assumes that stimulus satiation effects dissipate with time. The two-factor theory, assuming that only I_R develops when the responses are reinforced, would predict a decrease in turn alternation as the I_R dissipates. Since the S can alternate turns *or* stimuli but not both, any decrease in turn alternation must lead to an increase in the alternation of stimuli. Verification of this deduction was found in Thompson's (see Footnote 4) study; the percentage of turn alternation for 30-sec., 30-min., and 24-hr. distribution was 40, 29, and 28 respectively.

DEDUCTIONS RELEVANT TO EXTINCTION, REMINISCENCE, AND SPONTANEOUS RECOVERY

Deduction 7: Extinction, as measured by a decrease in spontaneous recovery, can be strengthened by continuing nonreinforced presentation of the CS after the CR has been extinguished to a point of nonelicitation.

This is sometimes referred to as *below zero extinction* and has been verified by Pavlov (1927) and Brogden, Lipman, & Culler (1938). According to the two-factor theory, both I_R and I_S develop during the extinction trials up to the point of nonelicitation of the CR with I_R dissipating after this point has been reached. Contin-

ued exposure of the S to the stimulus (CS) in the absence of reward would produce still more I_S which leads to a predicted decrease in spontaneous recovery.

Deduction 8: After rats have been trained to go to one side of a ⊤ maze, the number of trials to extinguish the response may be reduced by placing the S in the previously rewarded goal box without reward, prior to extinction proper.

According to the two-factor theory, I_S develops when the S is exposed to the stimulus in the absence of reward (as during latent extinction). Thus the number of extinction trials required are reduced. This deduction agrees with the experimental findings of Coate (1956), Deese (1951), Denny & Ratner (1959), Hurwitz (1955), Moltz (1955), Moltz & Maddi (1956), Seward & Levy (1949), and Thomas (1958).

Deduction 9: Spontaneous recovery will be reduced when latent extinction precedes experimental extinction, provided control and experimental Ss are given the same number of extinction trials.

The amount of I_R developed during experimental extinction is approximately the same for the two groups. However, the experimental group has more I_S, since some I_S was developed during the latent extinction period. Since I_S is assumed to dissipate at a very slow rate, the amount of spontaneous recovery for the experimental group should be reduced. Thomas (1958) found no significant difference between groups in amount of recovery, but his experimental design was such as to offer no direct test of the above deduction.

Deduction 10: When secondary reinforcement is minimized, continuously reinforced Ss will show superior performance to partially reinforced Ss at all stages of acquisition when the two groups are compared in terms of number of reinforced trials. The superiority of the 100% group will decrease with increase in intertrial interval.

The habit strength of both groups is equal since they have had the same number of reinforced trials. However, the amount of total inhibition is greater for the partially reinforced group since the total number of trials exceeds that of the 100% group. With an increase in intertrial interval the I_R dissipates for both groups, leaving the I_S developed on the nonreinforced trials for the partially reinforced group and thus decreasing the difference in performance. Verification of this deduction is seen in a recent study by Reynolds (1958) who found that when performance was compared in terms of number of reinforced trials, the curve for the 100% reinforced group remained significantly above that for the 60% reinforced group.

Deduction 11: The extinction curve of a continuously reinforced group will have a steeper initial slope than that of a partially reinforced group.

This deduction is based on the assumption that I_S develops according to a negative growth function of number of nonreinforced trials and dissipates very slowly. Since the partially reinforced group will have some I_S developed during the nonreinforced acquisition trials, the increment of I_S per trial will be smaller during the early extinction trials than for a continuously reinforced group. Verification of Deduction 11 is found in many studies, among them James & Rotter (1958), Reynolds (1958), Sheffield (1949), Weinstock (1954, 1958), and Wilson, Weiss, & Amsel (1955).

Deduction 12: Reminiscence will be complete for Ss trained under continuous reinforcements; i.e., following rest, the performance curve for a

massed group will be the same as the curve for a distributed group.

Since only I_R is assumed to develop on reinforced trials, reminiscence should be complete as I_R dissipates rapidly with the passage of time. Several studies on motor learning (Adams & Reynolds, 1954; Archer, 1954; Bilodeau, 1952; Schucker, Stevens, & Ellis, 1953; Starkweather & Duncan, 1954) have failed to verify Kimble's (1949) reported evidence for permanent work decrement on postrest performance following massed acquisition trials.

Deduction 13: Reminiscence will be incomplete for a long period of time following acquisition under partial reinforcement conditions; i.e., following rest, the performance curve for a partially reinforced group will be below the curve for a continuously reinforced group, provided the number of reinforced trials is held constant and secondary reinforcement is minimized. The amount of this "permanent" work decrement will be proportional to the number of nonreinforced trials.

This deduction is based on the assumption that I_S developed on the nonreinforced trials will dissipate very slowly, thus prolonging complete recovery of the performance curve.

Deduction 14: Spontaneous recovery will be less for a group which learned the response under partial reinforcement conditions than for a group which learned under continuous reinforcement, provided both groups are given the same number of extinction trials.

Total inhibition developed during extinction is the same for both groups, but the I_S resulting from the nonreinforced acquisition trials of the partially reinforced group yields a total amount of I_S exceeding that of the continuously reinforced group and results in less spontaneous recovery.

The author knows of no studies directly relevant to Deductions 13 and 14. However, several studies testing these and other deductions relevant to reminiscence and spontaneous recovery are underway in our laboratory.

SUMMARY

Because both the reactive inhibition and stimulus satiation explanations of spontaneous alternation seem inadequate, a two-factor theory of inhibition, in the form of three postulates, is presented. Deductions for the simple two-alternative situation, as well as extinction, reminiscence, and spontaneous recovery phenomena are listed, together with available supporting evidence.

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